Dynamical Electroweak Symmetry Breaking

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Motivations for Dynamical Electroweak Symmetry Breaking

There are several motivations for considering dynamical electroweak symmetry breaking (EWSB). Standard model (SM) Higgs mechanism is unsatisfying as an explanation:

Spontaneous symmetry breaking is put in by hand: Higgs potential $V(\phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$, $\mu^2 < 0$, so $\langle \phi \rangle = \binom{0}{v/\sqrt{2}}$, $\Rightarrow$ EWSB, but does not explain why μ^2 is negative when it could, a priori, have been positive.

μ^2 and hence $m_H^2 = -2\mu^2 = 2\lambda v^2$ are unstable to large radiative corrections from much higher energy scales (gauge hierarchy problem, fine-tuning needed to keep Higgs light).

The SM Yukawa mechanism for generating fermion masses, with $m_f \simeq y_f v/\sqrt{2}$, does not give insight into these masses, since it just puts in the values of Yukawa couplings y_f by hand, and some y_f's range down to 10^{-5} with no explanation.

Another important motivation: in two major previous cases where fundamental scalar fields were used to model spontaneous symmetry breaking, the actual underlying physics did not involve fundamental scalar fields but instead a bilinear fermion condensate:

Superconductivity: Ginzburg-Landau free energy functional used complex scalar field ϕ with $V = c_2|\phi|^2 + c_4|\phi|^4$, with $c_2 \propto (T - T_c)$, so for $T < T_c$, $c_2 < 0$ and $\langle\phi\rangle \neq 0$. But the actual physical origin of superconductivity is the dynamical formation of a condensate of Cooper pairs $\langle ee\rangle$ in BCS theory; since these are charged, they give mass to the photon, resulting in the Meissner effect, $|B(z)| \sim \exp(-m_\gamma z)$ in a superconductor.

Gell-Mann Lévy σ model for spontaneous chiral symmetry breaking (SχSB) in hadronic physics, with $V = (\mu^2/2)\vec{\phi}^2 + (\lambda/4)\vec{\phi}^4$, where $\vec{\phi} = (\sigma, \vec{\pi})$. In this model, one produces SχSB by the *ad hoc* choice $\mu^2 < 0$, leading to $\langle\sigma\rangle = f_\pi \neq 0$. However, the actual origin of SχSB in QCD is the dynamical formation of a $\langle\bar{q}q\rangle$ condensate.

The lesson from these previous phenomenological models of symmetry breaking is that the underlying physics responsible for this symmetry breaking may well be a dynamically induced fermion condensate.

Indeed, we already know of a source of dynamical EWSB: QCD breaks electroweak symmetry.

Consider, for simplicity, QCD with $N_f = 2$ massless quarks, u, d. This theory has a global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ chiral symmetry. The quark condensate $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R \rangle + \langle \bar{q}_R q_L \rangle$ transforms as an $I_w = 1/2$, $|Y| = 1$ operator and breaks this symmetry to the diagonal, vectorial isospin SU(2)$_V$. The resultant Nambu-Goldstone bosons (NGB's) - $\pi^{\pm}$ and π^0 - are absorbed to become the longitudinal components of the $W^{\pm}$ and Z, giving them masses:

$$m_W^2 = \frac{g^2 f_\pi^2}{4}\,, \qquad m_Z^2 = \frac{(g^2 + g'^2) f_\pi^2}{4}$$

With $f_\pi \sim 90$ MeV, this yields $m_W \simeq 30$ MeV, $m_Z \simeq 33$ MeV. These masses satisfy the tree-level relation $\rho = 1$, where

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W}$$

While the scale here is too small by $\sim 10^3$ to explain the observed W and Z masses, it suggests how to construct a model with dynamical EWSB:

Basics of Dynamical EWSB

Consider an asymptotically free vectorial gauge theory ("technicolor", TC) with gauge group $\mathrm{SU}(N_{TC})$, with a set of fermions F with zero Lagrangian masses, transforming according to some representation of G; the TC interaction becomes strong at a scale Λ_{TC} of order the electroweak scale, confining and producing a chiral symmetry breaking technifermion condensate (Weinberg, Susskind, 1979; recent review: Sannino, ArXiv:0911.0931).

Assign technifermions so L (R) components form SU(2)$_L$ doublets (singlets). Minimal choice: "one-doublet" (1DTC) model, uses

$$\begin{pmatrix} F_u^\tau \\ F_d^\tau \end{pmatrix}_L \qquad F_{uR}^\tau, \quad F_{dR}^\tau$$

with TC indices τ and $Y = 0$ ($Y = \pm 1$) for SU(2)$_L$ doublet (singlets).

The TC theory is asymptotically free, so as energy scale decreases, α_{TC} increases, eventually producing condensates $\langle \bar{F}_u F_u \rangle$, $\langle \bar{F}_d F_d \rangle$ transforming as $I_w = 1/2$, $|Y| = 1$, breaking EW symmetry at Λ_{TC}.

Just as in the QCD example above, the W and Z pick up masses, but now involving the TC scale:

$$m_W^2 \simeq \frac{g^2 F_{TC}^2 N_D}{4}, \qquad m_Z^2 \simeq \frac{(g^2 + g'^2) F_{TC}^2 N_D}{4}$$

again satisfying the tree-level relation $\rho = 1$. Here $F_{TC} \sim \Lambda_{TC}$ is the TC analogue to $f_\pi \sim \Lambda_{QCD}$ and $N_D =$ number of SU(2)$_L$ technidoublets. For this minimal example, $N_D = 1$, so $F_{TC} = 250$ GeV.

Another class of TC models uses one SM family of technifermions (1FTC)

$$\begin{pmatrix} U^{a\tau} \\ D^{a\tau} \end{pmatrix}_L \quad U_R^{a\tau}, \quad D_R^{a\tau}$$

$$\begin{pmatrix} N^{\tau} \\ E^{\tau} \end{pmatrix}_L \quad N_R^{\tau}, \quad E_R^{\tau}$$

(a, τ color, TC indices) with usual Y assignments. Similar condensate formation, with approx. equal condensates $\langle \bar{F} F \rangle$ for $F = U^a,\ D^a,\ N,\ E$, generating dynamical technifermion masses $\Sigma_{TC} \sim \Lambda_{TC}$, analogous to constituent quark mass $\sim \Lambda_{QCD}$ in QCD. Resultant m_W^2 and m_Z^2 given by formula above with $N_D = N_c + 1 = 4$, so $F_{TC} = 125$ GeV for 1FTC.

Many models have been studied where technifermions transform according to the fundamental representation of G_{TC}. There has also been much work by Sannino, Tuominen, Dietrich, Ryttov, Kouvaris, Frandsen, Gudnason, and also by lattice groups for technifermions transforming according to higher representations of G_{TC}.

TC has the potential to solve/explain problematic/mysterious features of SM:

- Given the asymptotic freedom of the TC theory, the condensate formation and hence EWSB are automatic and do not require any ad hoc parameter choice like $\mu^2 < 0$ in the SM.

- Because TC has no fundamental scalar field, there is no hierarchy problem.

- Because $\langle \bar{F} F \rangle = \langle \bar{F}_L F_R \rangle + \langle \bar{F}_R F_L \rangle$, technicolor explains why the chiral part of G_{SM} is broken and the residual exact gauge symmetry, $\mathrm{SU}(3)_c \times \mathrm{U}(1)_{em}$, is vectorial (also explained with SM Higgs).

To give masses to quarks and leptons, must communicate the EWSB in the TC sector to these SM fermions; hence, embed TC in a larger, extended technicolor (ETC) gauge theory with ETC gauge bosons V_τ^i transforming (technisinglet) SM fermions into technifermions (Dimopoulos and Susskind; Eichten and Lane, 1979-80) (other sources of SM fermion masses such as topcolor are possible; we focus here on ETC).

To satisfy constraints on flavor-changing neutral current (FCNC) processes, ETC gauge bosons must have large masses. These masses can arise from self-breaking (tumbling) of the ETC chiral gauge symmetry. ETC theory is arranged to be asymptotically free, so as energy decreases from high scale, ETC coupling α_{ETC} grows, eventually becomes large enough to form condensates which sequentially break the ETC symmetry group down at scales Λ_j, $j = 1, 2, 3$ to a residual exact TC subgroup, with, e.g.

$$\Lambda_1 \simeq 10^3 \text{ TeV}, \qquad \Lambda_2 \simeq 50 - 100 \text{ TeV}, \qquad \Lambda_3 \simeq \text{few TeV},$$

We will focus on ETC models with one-family TC; gauge the SM fermion generation index and combine it with the TC index τ. Hence, for an SU(N_{ETC}) ETC theory,

$$N_{ETC} = N_{gen.} + N_{TC} \ = \ 3 + N_{TC}$$

The choice $N_{TC} = 2$ is motivated for several reasons:

- minimizes the TC contributions to precision electroweak quantities
- can yield an approximate IR fixed point and resultant slow running of TC coupling
- makes possible a mechanism for explaining neutrino masses in an ETC context

$N_{gen.} = 3$, $N_{TC} = 2 \implies N_{ETC} = 5$, so SU(5)$_{ETC}$ theory (early example: Appelquist and Terning, 1994).

Progress on Understanding Walking Gauge Theories

To be viable, modern TC models are designed to have a coupling g_{TC} that gets large, but runs slowly ("walks") over an extended interval of energy (WTC) (Holdom, Yamawaki et al., Appelquist et al.):

$$\beta(\alpha_{TC}) = \frac{d\alpha_{TC}}{dt} = -\frac{\alpha_{TC}^2}{2\pi}\left(b_1 + \frac{b_2\,\alpha_{TC}}{4\pi} + O(\alpha_{TC}^2)\right)$$

where $t = \ln\mu$, with $b_1 > 0$ - asymp. freedom. For sufficiently many technifermions, $b_2 < 0$, so β has a second zero (approximate IR fixed point of RG) at $\alpha_{TC} = -4\pi b_1/b_2 \equiv \alpha_{IR}$. To analyze the properties of the theory, study the Dyson-Schwinger (DS) equation for the fermion propagator; for $\alpha > \alpha_{cr}$, this yields a nonzero solution for a dynamically generated fermion mass (βDS method). Simple ladder approx. to DS eq. gives $\alpha_{cr}C_2(R) \sim O(1)$, where R is fermion rep.

As number of technifermions, N_f, increases, α_{IR} decreases. In WTC, arrange so that α_{IR} is slightly greater than the critical value, α_{cr}, for $\langle \bar{F}F\rangle$ formation. As $N_f \nearrow N_{f,cr}$, $\alpha_{IR} \searrow \alpha_{cr}$. Calculations with Dyson-Schwinger (DS) eq. suggest that for $N_{TC} = 2$ and technifermions in the fundamental rep. of SU(2)$_{TC}$, $N_{f,cr} \simeq 8$. So one-family TC, which has $N_f = N_w(N_c+1) = 8$ technifermions, plausibly exhibits walking behavior. Another possibility: SU(2)$_{TC}$, with 2 adjoint technifermions making up one SU(2)$_L$–doublet + RH SU(2)$_L$ singlets (Sannino et al.).

Then as energy scale decreases, $\alpha_{_{TC}}$ grows, but its rate of increase, $|\beta|$, decreases toward zero as $\alpha_{_{TC}} \nearrow \alpha_{_{IR}}$, where $\beta = 0$. Eventually, $\alpha_{_{TC}}$ exceeds α_{cr}, $\langle \bar{F} F \rangle$ forms, breaking EW symmetry, and technifermions gain dynamical masses $\Sigma_{TC} \simeq \Lambda_{TC}$. In low-energy effective field theory below Λ_{TC}, β function coeffs. hence change, so this is an approx. IR fixed point.

Walking TC has several advantages: (i) SM fermion masses are enhanced by the factor

$$\eta = \exp\left[\int_{\Lambda_{TC}}^{\Lambda_w} \frac{d\mu}{\mu}\,\gamma(\alpha_{_{TC}}(\mu))\right]$$

where the TC theory has walking up to Λ_w (typically Λ_3); if $\gamma \simeq 1$, this yields $\eta \simeq \Lambda_w/\Lambda_{TC}$; (ii) hence, one can increase ETC scales Λ_i for a fixed m_{f_i}, reducing FCNC effects; (iii) η also can enhance masses of pseudo-Nambu Goldstone bosons (PNGB's); (iv) can reduce electroweak S parameter.

For model-building, it is important to verify that, with specified technifermion contents, a given model exhibits walking behavior. Recently, there has been considerable progress in understanding walking gauge theories.

An example is the estimate from the beta function calculation of α_{IR} combined with the DS eq. that $N_{f,cr} \simeq 8$ for an SU(2)$_{TC}$ theory with technifermions in the fundamental rep. The solution of the DS equation yields a nonzero value of the dynamically generated technifermion mass and hence $\langle \bar{F}F \rangle$ for the approx. IR fixed-point value $\alpha > \alpha_{cr}$, and hence $N_f < N_{f,cr}$.

We know that there can be significant higher-order corrections to the estimate from this βDS method of calculating $N_{f,cr}$, in view of the strongly-coupled nature of the physics. The βDS method captures some of the relevant physics, but does not directly include the effects of (i) confinement or (ii) instantons. Both of these are important for spontaneous chiral symmetry breaking.

Lattice gauge simulations can provide a fully nonperturbative determination of $N_{f,cr}$ and provide a check on the βDS method. Although TC favors an SU(2) gauge group, much recent lattice work has understandably been for SU(3) gauge theory, since for SU(3) one can take advantage of extensive experience and computer codes from lattice QCD calculations, and then study how varying N_f changes the properties of the theory.

For SU(3), with (massless) fermions in the fundamental rep., the βDS method gives $N_{f,cr} \simeq 12$. Recent lattice-based studies on the (zero-temp.) phase structure of SU(3) gauge theory as a function of N_f by Appelquist, Fleming, Neil, 2008, 2009 and LSD Collab.; Deuzeman, Lombardo, Pallante, 2008, 2009; Jin and Mawhinney 2009; Fodor, Holland, Kuti, Nogradi, Schroeder 2008, 2009; Hasenfratz 2009; Yamada et al. 2009. Although there are some differences in results from various groups, they find rough agreement with the predictions from the βDS method, to within the uncertainties inherent in that method.

Also several recent studies of $N_{f,cr}$ for fermions in higher representations, e.g., for SU(2) with adjoint fermions and/or SU(3) with sextet fermions; studies by Catterall and Sannino; Catterall, Giedt, Sannino, Schneible; DeGrand, Shamir, Svetitsky; DeGrand, Del Debbio et al., Fodor, Holland, Kuti, Nogradi, Schroeder, and others.

It is interesting that the DS analysis appears to give reasonably accurate results, even though it neglects higher-order technigluon exchanges and instanton contributions, which enhance SχSB.

We can understand how the DS prediction for $N_{f,cr}$ can nevertheless be roughly correct (Brodsky and RS, PL B666, 95 (2008), ArXiv:0806.1535). Both QCD and TC are confining theories. Because of confinement, the quarks and gluons in QCD (resp. techniquarks and technigluons in TC) have maximum wavelengths and hence minimum momenta $k_{min} \simeq \Lambda_{QCD}$ for QCD, $k_{min} \simeq \Lambda_{TC}$ for TC.

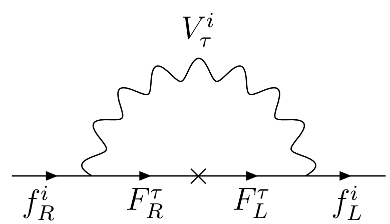
This has important implications. In QCD, this can explain an old question about the argument (Appelquist-Politzer, 1974) for the narrow width of a ${}^3S_1$ heavy $Q\bar{Q}$ state (J/ψ, etc.); these states are narrow because $Q\bar{Q}$ annihilation to 3 gluons involves small α, due to asymptotic freedom and $m_Q >> \Lambda_{QCD}$. But what about emission into a large number n of gluons, with each carrying $2m_Q/n$ momentum, with $\alpha(2m_Q/n)$ not small? Our answer: such emission is kinematically forbidden because of the minimum gluon momentum, k_{min}.

Since the DS eqs. do not incorporate the property of confinement, they take the lower end of the Euclidean loop integration interval to momentum $k \to 0$. But because of confinement, the quarks and gluons (technifermions and technigluons) have nonzero bound state momenta of order Λ_{QCD} (Λ_{TC}, resp.), so the k integration should not really extend all the way to 0; the k integration measure is smaller.

Hence, two major aspects in which the DS analysis neglects relevant physics go in opposite directions: (i) the neglect of instantons produces an underestimate of SχSB, and (ii) the neglect of confinement and resultant use of a larger loop integration interval than is the case produces an overestimate of SχSB. Since the effects of these different physical features that are neglected are opposite, the simple DS analysis may thus be able to get a roughly correct answer for $N_{f,cr}$.

Mass Generation Mechanism for Fermions

The ETC gauge bosons enable SM fermions, which are TC singlets, to transform into technifermions and back. This provides a mechanism for generating SM fermion masses. The figure shows a one-loop graph contributing to diagonal entries in mass matrix for SM fermion f^i. Basic ETC vertex is $f^i \longrightarrow f^j + V^i_j$, with $V^i_j =$ ETC gauge boson, $1 \le i, j \le 5$; here we distinguish the first three ETC indices, which refer to SM fermion generations, and the indices 4,5 which are the SU(2)$_{TC}$ indices, by denoting the latter as $\tau = 4, 5$.



Rough estimate:

$$M^{(f)}_{ii} \simeq \frac{2\alpha_{ETC} C_2(R)}{\pi} \int dk^2 \frac{k^2 \Sigma_{TC}(k^2)}{[k^2 + \Sigma_{TC}(k^2)^2][k^2 + M_i^2]}$$

where $M_i \simeq (g_{ETC}/4)\Lambda_i \simeq \Lambda_i$ is the mass of the ETC gauge bosons that gain mass at scale Λ_i, $C_2(R) =$ quadratic Casimir invariant. With walking technicolor,

$\Sigma_{TC}(k) \simeq \Sigma_{TC}(0)^2/k$ for Euclidean $k \gg \Lambda_{TC}$ in walking regime (contrast with QCD, where $\Sigma(k) \simeq \Sigma(0)^3/k^2$ for $k \gg \Lambda_{QCD}$), this gives

$$M_{ii}^{(f)} \simeq \frac{\kappa\, C_2(R)\, \eta\, \Lambda_{TC}^3}{\Lambda_i^2}$$

where $\kappa \simeq O(10)$ is a numerical factor from the integral and in WTC, $\eta \simeq \Lambda_3/\Lambda_{TC}$. This is only a rough estimate, since ETC coupling is strong, so higher-order diagrams are also important.

The sequential breaking of the ETC symmetry at the highest scale Λ_1, the intermediate scale Λ_2, and the lowest scale Λ_3 thus produces the generational hierarchy in the fermion masses. Since these ETC scales enter as inverse powers in the resultant SM fermion masses and since Λ_1 is the largest ETC scale, it follows that first-generation fermion masses are the smallest, and since Λ_3 is the smallest ETC scale, third-generation fermion masses are the largest.

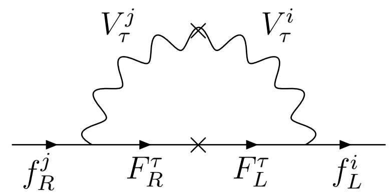
From this analysis, one can infer the asymptotic momentum dependence of the running mass $m_{f_i}(p)$ of a SM fermion of generation i (Christensen and RS, Phys. Rev. Lett. 94, 241801 (2005)): $m_{f_i}(p)$ is constant up to $M_i \simeq \Lambda_i$ and has the power-law decay

$$m_{f_i}(p) \sim m_{f_i(0)} \frac{\Lambda_i^2}{p^2}$$

for Euclidean momenta $p \gg \Lambda_i$. Here we neglect logarithmic factors, which are subdominant relative to this power-law falloff.

Thus, $m_t(p)$ and $m_b(p)$ decay like Λ_3^2/p^2 for $p \gg \Lambda_3$, while $m_u(p)$ and $m_d(p)$ are hard up to the much higher scale Λ_1.

There are mixings among the interaction eigenstates of the ETC gauge bosons to form mass eigenstates. These involve mixings $V_\tau^j \to V_\tau^i$, where $i, j \in \{1, 2, 3\}$ and $\tau \in \{4, 5\}$. Insertions of these on ETC gauge boson lines lead to off-diagonal elements of the $M^{(f)}$ via diagrams like



This yields

$$M_{ij}^{(f)} \simeq \frac{\kappa\, \eta\, \Lambda_{TC}^3\, {}^j_\tau\Pi^i_\tau}{\Lambda_i^2 \Lambda_j^2}$$

where ${}^j_\tau\Pi^i_\tau$ is the nondiagonal vacuum polarization tensor function representing the ETC gauge boson mixing $V_\tau^i \to V_\tau^j$. Also direct contributions to $M_{33}^{(f)}$ from exchange of V_{d3}, where $T_{d3} \propto \mathrm{diag}(0, 0, -2, 1, 1)$ which could be significant. Diagonalization of the full SM fermion mass matrices yields masses and mixings.

ETC Model and its Properties

One gains considerable insight from the construction and study of reasonably ultra-violet complete ETC models. We illustrate this with a study of a model in which one-family SU(2)$_{TC}$ is embedded in an SU(5) ETC theory, where $N_{ETC} = N_{gen.} + N_{TC}$ (Appelquist, Piai, RS, Phys. Rev. D 69, 015002 (2004); Phys. Lett. B 593, 175 (2004); Phys. Lett. B 595, 442 (2004); Appelquist, Christensen, Piai, RS, Phys. Rev. D 70, 093010 (2004)). Consider an ETC model with gauge group

$$G = \mathrm{SU}(5)_{ETC} \times \mathrm{SU}(2)_{HC} \times G_{SM}$$

(Hypercolor (HC) is used for sequential ETC symmetry breaking.) Fermions:

$$Q_L : \ (5,1,3,2)_{1/3,L} \ , \qquad u_R : \ (5,1,3,1)_{4/3,R} \ , \qquad d_R : \ (5,1,3,1)_{-2/3,R} \ ,$$

$$L_L : \ (5,1,1,2)_{-1,L}, \qquad e_R : \ (5,1,1,1)_{-2,R} \ ,$$

$$\psi_{ij,R} : \ (\overline{10},1,1,1)_{0,R} \ , \qquad \zeta_R^{ij,\alpha} : \ (10,2,1,1)_{0,R} \ , \qquad \omega^{\alpha}_{p,R} : \ 2(1,2,1,1)_{0,R}$$

where here $1 \le i,j \le 5$ are SU(5)$_{ETC}$ indices, $\alpha = 1,2$ is an SU(2)$_{HC}$ index, and $p = 1,2$ is a copy number.

We use compact notation $f = u,\ d,\ e$, e.g. (with $a =$ color index) so, e.g.,

$$u_R \ : (u^{1,a}, u^{2,a}, u^{3,a}, u^{4,a}, u^{5,a})_R \ \equiv \ (u^a, c^a, t^a, U^{4,a}, U^{5,a})_R$$
$$d_R \ : (d^{1,a}, d^{2,a}, d^{3,a}, d^{4,a}, d^{5,a})_R \ \equiv \ (d^a, s^a, b^a, D^{4,a}, D^{5,a})_R$$
$$e_R \ : (e^1, e^2, e^3, e^4, e^5)_R \ \equiv \ (e, \mu, \tau, E^4, E^5)_R$$

The SU(5)$_{ETC}$ group has vectorial couplings to SM fermions, techniquarks, and charged technileptons, but the SM-singlet sector makes the full theory a chiral gauge theory. No fermion mass terms in $\mathcal{L}$. The theory has no gauge or global anomalies.

Next, analyze how the sequential breaking of SU(5)$_{ETC}$ to the exact residual strongly coupled gauge symmetry SU(2)$_{TC}$ occurs.

Since the ETC theory is asymptotically free, its gauge coupling grows as the energy scale decreases. The ETC breaking occurs because of the formation of ETC-noninvariant bilinear condensates of ETC-nonsinglet, SM-singlet fermions.

We identify likely condensation channels and vacuum alignment by use of a generalized most attractive channel (MAC) criterion. Approximate measure of attractiveness of a channel $R_1 \times R_2 \to R_{cond.}$ is $\Delta C_2 = C_2(R_1) + C_2(R_2) - C_2(R_{cond.})$ where $C_2(R)$ is the Casimir for the rep. R. Expect condensation when $\alpha_{ETC} \Delta C_2(R) \simeq O(1)$.

Envision that at scale $\Lambda_1 \simeq 10^3$ TeV, α_{ETC} becomes large enough to produce condensation in channel

$$(\overline{10}, 1, 1, 1)_{0,R} \times (\overline{10}, 1, 1, 1)_{0,R} \to (5, 1, 1, 1)_0$$

with $\Delta C_2 = 24/5$, breaking SU(5)$_{ETC}$ to SU(4)$_{ETC}$. With no loss of generality, take breaking direction as $i = 1$; this entails the separation of the first generation SM fermions from the other components of their respective ETC multiplets with $2 \le i \le 5$.

The nine ETC gauge bosons in coset $\mathrm{SU}(5)_{ETC}/SU(4)_{ETC}$ gain masses $M_1 \simeq \Lambda_1$, viz., V^1_j, $(V^1_j)^\dagger = V^j_1$, with $j = 2, .., 5$, and V_{d1} corresponding to generator $T_{24} \propto \mathrm{diag}(-4, 1, 1, 1, 1)$.

Since the effective field theory below Λ_1 has SU(4)$_{ETC}$ symmetry, to analyze this, we decompose the SU(5)$_{ETC}$ representations in terms of SU(4)$_{ETC}$ reps. Among SM singlets, the $\overline{10}$ of SU(5)$_{ETC}$ decomposes as $\bar{4} + \bar{6}$

Denote

$$(\bar{4}, 1, 1, 1)_{0,R} = \psi_{1i,R} \equiv \alpha_{1i,R}\,, \qquad (\bar{6}, 1, 1, 1)_{0,R} = \psi_{ij,R} \equiv \xi_{ij,R}\,, \quad 2 \le i, j \le 5$$

The associated condensate is

$$\langle\epsilon^{1ijk\ell}\xi^T_{ij,R}C\xi_{k\ell,R}\rangle = 8\langle\xi^T_{23,R}C\xi_{45,R} - \xi^T_{24,R}C\xi_{35,R} + \xi^T_{25,R}C\xi_{34,R}\rangle$$

This evidently transforms as a 5 of SU(5)$_{ETC}$. The six fields $\xi_{ij,R}$, $2 \le i,j \le 5$, involved in this condensate gain dynamical masses $\simeq \Lambda_1$. (The mass eigenstates are linear combinations of these fields.)

For the SM fermions, the decomposition is $5_{SU(5)} = (4+1)_{SU(4)}$:

Q_L : $(5,1,3,2)_{1/3,L}$ breaks into the fund. rep. of SU(4)$_{ETC}$,
Q_L : $(4,1,3,2)_{1/3,L}$ and the SU(4)$_{ETC}$ singlet $\binom{u}{d}_L$: $(1,1,3,2)_{1/3,L}$,

u_R : $(5,1,3,1)_{4/3,R}$ breaks into $(4,1,3,1)_{4/3,R}$ and $(1,1,3,1)_{4/3,R}$, etc. for d_R, L_L, and e_R.

At energy scales below Λ_1, depending on relative strengths of gauge couplings, different symmetry-breaking sequences can occur. SU(4)$_{ETC}$ and SU(2)$_{HC}$ are asymptotically free, so their gauge couplings grow further as E decreases.

One sequence we call S_1: at $E = \Lambda_2 \simeq 50 - 100$ TeV, SU(4)$_{ETC}$ and SU(2)$_{HC}$ couplings are sufficiently large to lead together to the condensation

$$(4,2,1,1)_{0,R} \times (6,2,1,1)_{0,R} \to (\bar{4},1,1,1)_0$$

(with $\Delta C_2 = 5/3$ for SU(4)$_{ETC}$ and $\Delta C_2 = 3/2$ for SU(2)$_{HC}$). This breaks SU(4)$_{ETC}$ to SU(3)$_{ETC}$. With no loss of generality, define the broken direction as $i = 2$; then the second-generation SM fermions split off from the remaining $i = 3, 4, 5$ components of their ETC multiplets.

The condensate is

$$\langle \epsilon_{\alpha\beta}\epsilon_{12jk\ell}\zeta_R^{1j,\alpha\ T} C \zeta_R^{k\ell,\beta}\rangle = 2\langle \epsilon_{\alpha\beta}(\zeta_R^{13,\alpha\ T} C\zeta_R^{45,\beta} - \zeta_R^{14,\alpha\ T} C\zeta_R^{35,\beta} + \zeta_R^{15,\alpha\ T} C\zeta_R^{34,\beta})\rangle \ ,$$

and (linear combinations of) the twelve $\zeta_R^{ij,\alpha}$ fields in this condensate gain masses $\simeq \Lambda_2$. The seven ETC gauge bosons in the coset $\mathrm{SU}(4)_{ETC}/\mathrm{SU}(3)_{ETC}$, viz., V_j^2, V_2^j with $j = 3, 4, 5$, and V corresponding to generator $T_{15} \propto \mathrm{diag}(0, -3, 1, 1, 1)$, gain masses $M_2 \simeq \Lambda_2$.

Since the effective field theory below Λ_2 has SU(3)$_{ETC}$ symmetry, to analyze this, we decompose the SU(4)$_{ETC}$ representations in terms of SU(3)$_{ETC}$.

The resultant $\mathrm{SU}(3)_{ETC} \times \mathrm{SU}(2)_{HC}$ strongly coupled gauge symmetry operative below Λ_2 is asymp. free, so couplings continue to grow, and at the lowest ETC scale,

$\Lambda_3 \simeq 3$ TeV, the $(3,2,1,1)_{0,R}$, $\zeta_R^{2j,\alpha}$, $j=3,4,5$ from the previous SU(4)$_{ETC}$ rep. $(6,2,1,1)_{0,R}$ condenses as

$$(3,2,1,1)_{0,R} \times (3,2,1,1)_{0,R} \to (\bar{3},1,1,1)_0$$

(with $\Delta C_2 = 4/3$ for SU(3)$_{ETC}$ and $\Delta C_2 = 3/2$ for SU(2)$_{HC}$) breaking SU(3)$_{ETC}$ to SU(2)$_{TC}$. The condensate is

$$\langle \epsilon_{123ij}\epsilon_{\alpha\beta}\zeta_R^{2i,\alpha\ T} C \zeta_R^{2j,\beta}\rangle \ = \ 2\langle \zeta_R^{24,1\ T} C \zeta_R^{25,2} - \zeta_R^{24,2\ T} C \zeta_R^{25,1}\rangle$$

The five ETC gauge bosons in the coset $\mathrm{SU}(3)_{ETC}/\mathrm{SU}(2)_{TC}$, viz., V_j^3, V_3^j with $j=4,5$, and V corresponding to generator $T_8 \equiv T_{d3} \propto \mathrm{diag}(0,0,-2,1,1)$, gain masses $M_3 \simeq \Lambda_3$. At this scale Λ_3, the third generation of SM fermions splits off, leaving the residual technifermions in each of the respective multiplets.

At a comparable scale $\simeq \Lambda_3$, other condensates respecting the same symmetries form, including

$$\langle \epsilon_{\alpha\beta}\zeta_R^{12,\alpha\ T} C \zeta_R^{23,\beta}\rangle\,, \quad \langle \epsilon_{\alpha\beta}\zeta_R^{12,\alpha\ T} C \omega_{p,R}^{\beta}\rangle\,, \quad \langle \epsilon_{\alpha\beta}\zeta_R^{23,\alpha\ T} C \omega_{p,R}^{\beta}\rangle\,, \quad \langle \epsilon_{\alpha\beta}\omega_{1,R}^{\alpha\ T} C \omega_{2,R}^{\beta}$$

with $p = 1, 2$ and the fermions involved in these condensates gain dynamical masses of order Λ_3.

At energy scales below Λ_3, the SU(2)$_{TC}$ coupling continues to grow and finally produces the technifermion condensates at $\Lambda_{TC} \simeq 300$ GeV, breaking electroweak symmetry.

We have also studied an alternate ETC symmetry-breaking sequence, called S_2, which could be preferred for a smaller α_{HC} wrt. α_{ETC}; as E decreases to $\Lambda_{BHC} \lesssim \Lambda_1$ (BHC = broken HC), the SU(4)$_{ETC}$ interaction produces a condensation $(6, 2, 1, 1)_{0,R} \times (6, 2, 1, 1)_{0,R} \to (1, 3, 1, 1)_0$.

With respect to SU(4)$_{ETC}$, $\Delta C_2 = 5$; this occurs at a lower scale than Λ_1 because it is repulsive with respect to HC ($\Delta C_2 = -1/4$). The condensate is

$$\langle \epsilon_{1ijk\ell} \zeta_R^{ij,1\ T} C \zeta_R^{k\ell,2} \rangle + (1 \leftrightarrow 2)$$

This is an adjoint rep. of hypercolor and breaks $\mathrm{SU}(2)_{HC} \to \mathrm{U}(1)_{HC}$. Let $\alpha = 1, 2$ correspond to $Q_{HC} = \pm 1$ under the U(1)$_{HC}$. The twelve $\zeta_R^{ij,\alpha}$ fields involved gain dynamical masses $\simeq \Lambda_{BHC}$.

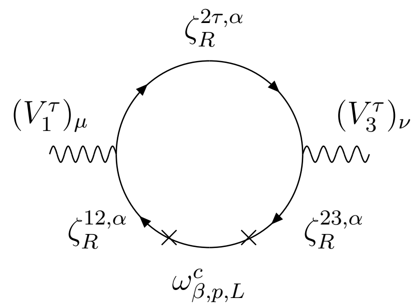
At the lower scale, Λ_{23}, the SU(4)$_{ETC}$ and U(1)$_{HC}$ interactions produce the condensation $4 \times 4 \to 6$ with $\Delta C_2 = 5/4$ and condensate $\langle \epsilon_{\alpha\beta} \zeta_R^{12,\alpha\ T} C \zeta_R^{13,\beta} \rangle$, which breaks $\mathrm{SU}(4)_{ETC} \to \mathrm{SU}(2)_{ETC}$ and is U(1)$_{HC}$-invariant. We take $\Lambda_{23} \simeq 50$ TeV. Other condensates also form at this scale:

$$\langle \zeta_R^{1i,\alpha\ T} C \omega_{p,R}^{\beta} \rangle,\ i = 2,3,\ p = 1,2, \quad \langle \omega_{p,R}^{\alpha\ T} C \omega_{p',R}^{\beta} \rangle,\ p, p' = 1,2, \alpha \neq \beta$$

The U(1)$_{HC}$ interaction does not couple directly to SM particles.

Finally, at $E \simeq \Lambda_{TC}$, technifermion condensation occurs, breaking electroweak symmetry.

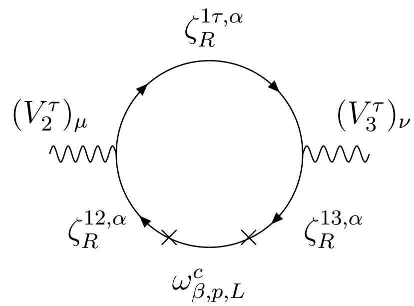
The fermion condensates contribute to nondiagonal ETC gauge boson propagator corrections and hence mixing. For example, in sequence S_1, the mixing $V_1^\tau \to V_3^\tau$, $\tau = 4, 5$ is induced by



$$ {}_3^{\tau}\Pi_1^{\tau}(0) \simeq \frac{\alpha_{ETC}}{4\pi} \int (k^2 dk^2) \frac{k^4\, \Sigma_3(k)^2}{[k^2 + \Sigma_3(k)^2]^4} $$

where $\Sigma_3 \simeq \Lambda_3$. This yields ${}_3^{\tau}\Pi_1^{\tau}(0) \simeq \text{const.} \times \Lambda_3^2$.

In sequence S_2, the mixing $V_2^\tau \to V_3^\tau$, $\tau = 4, 5$ is induced by



giving ${}_3^\tau\Pi_2^\tau(0) \simeq \mathrm{const.} \times \Lambda_{23}^2$.

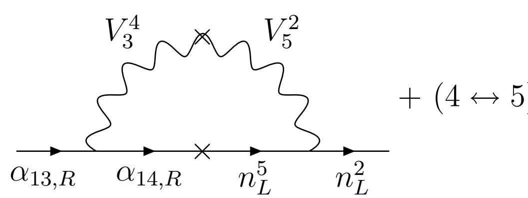
We find that the feature that nondiagonal ETC gauge boson propagator corrections ${}_i^\tau\Pi_j^\tau(0)$ are proportional to the square of the lowest ETC scale (or smaller) is generic in this type of ETC model, reflecting a type of approximate generational symmetry. Other ETC gauge boson mixings are similarly suppressed. Further corrections to $M^{(f)}$ arise from SM gauge interactions, in particular, SU(3)$_c$ and U(1)$_Y$, and from direct diagonal ETC exchanges, in particular, V_{d3}.

Although this ETC model is not fully realistic, it shows how not just diagonal but also off-diagonal elts. of SM fermion mass matrices $M^{(f)}$ and hence CKM mixing, could arise dynamically. Weak hypercharge can produce differences between mixing in the up and down quark sectors, which are generically magnified in a walking theory. Also, color attraction with similar magnification due to walking could make techniquark condensates and dynamical masses somewhat larger than those of technileptons.

It was a longstanding problem to explain light neutrino masses in a TC/ETC theory since this has no GUT mass scale. However, we have shown how such masses can arise in the present type of model (Appelquist and RS, Phys. Lett. B 548, 204 (2002); Phys. Rev. Lett. 90, 201801 (2003)).

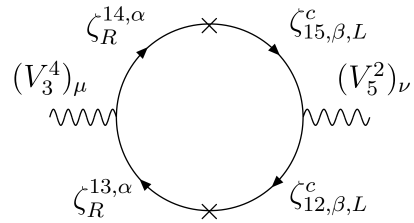
The states $\alpha_{1j,R}$ with $j = 2, 3$ are RH EW-singlet neutrinos and get induced Dirac neutrino mass terms connecting with $(n^1, n^2, n^3)_L = (\nu_e, \nu_\mu, \nu_\tau)_L$. These Dirac masses $\bar{n}_{i,L} M_D \alpha_{1j,R}$ cannot be generated by the usual one-loop ETC graphs that produce diagonal quark and charged lepton masses and are thus suppressed. Denote $(M_D)_{ij} \equiv b_{ij}$, $i = 1, 2, 3$, $j = 2, 3$.

Example of a graph contributing to b_{23} in sequence S_2:

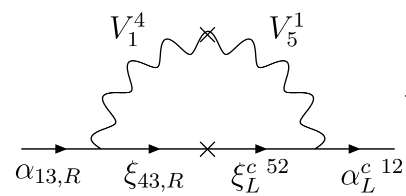


where higher-order exchanges of ETC gauge bosons are understood, since ETC is strongly coupled.

The indicated ETC gauge boson mixing involves



This yields as dominant terms $b_{ij} \sim \Lambda_{TC}^4/\Lambda_{23}^3 \simeq O(0.1)$ MeV, for $2 \le i, j \le 3$. The $\alpha_{1j,R}$ also have induced Majorana mass terms $\alpha^T_{1i,R} C r_{ij} \alpha_{1j,R}$. Graphs contributing to r_{23}:



With walking up to Λ_3, these yield $r_{23} \propto \Lambda^3_{BHC}\Lambda^3_{23}/\Lambda^5_1$. Numerically, $|r_{23}| \simeq O(10^2)$ GeV.

The resultant EW-nonsinglet neutrinos are, to very good approximation, linear combinations of three mass eigenstates, with normal hierarchy and ν_3 mass

$$m(\nu_3) \simeq \frac{(b_{23}+b_{22})^2}{r_{23}} \simeq \frac{\Lambda_{TC}^8 \Lambda_1^5}{\Lambda_{23}^9 \Lambda_{BHC}^3}$$

With the above-mentioned numerical values and $\Lambda_{BHC} \simeq 0.3\Lambda_1$, we find $m(\nu_3) \simeq 0.05$ eV, consistent with experimental results on neutrino oscillations. For ν_2:

$$\frac{m(\nu_2)}{m(\nu_3)} = \left(\frac{b_{23}-b_{22}}{b_{23}+b_{22}}\right)^2$$

consistent with data; also, model predicts $m(\nu_1) \ll m(\nu_2),\ m(\nu_3)$.

Since $|r_{23}| >> |b_{ij}|$, this is a seesaw, but quite different from the SUSY GUT seesaw; the Majorana masses r_{ij} that underly the seesaw are not GUT-scale and are actually much smaller than the ETC scales Λ_i. Because of the strong-coupling nature of the physics, it is difficult to give precise predictions for neutrino mixing angles. The neutrino mass eigenstates ν_h with masses $\sim r_{23}$ are mainly comprised of $\alpha_{12,R}$ and $\alpha_{13,R}$ but have small admixtures of ν_ℓ, with coefficients $\theta_{\nu_h \nu_\ell} \sim m_D/m_R \simeq b_{ij}/r_{23} \simeq 10^{-5} - 10^{-6}$.

Other Constraints on, and Phenomenology of ETC models

In the SM, MSSM, and GUT's, generations are put in by hand and are not related to gauge structure. ETC is quite different since it gauges generations. This is natural in order to communicate EWSB in the TC sector to the SM fermions. It gives rise to FCNC processes and is constrained by the limits or data on these.

An early attitude was that FCNC constraints were very severe for ETC theories and that the ETC scale(s) had to be so high that there would be excessive suppression of SM fermion masses. However, working with a detailed ETC model, we have shown that this early attitude was overly pessimistic.

For example, consider $K^0 - \bar{K}^0$ mixing and resultant $K_L - K_S$ mass difference $\Delta m_{K_L K_S}$. SM contribution consistent with experimental value $\Delta m_{K_L K_S}/m_K \simeq 0.7 \times 10^{-14}$.

Naive effective Lagrangian used in early studies without UV-complete ETC model: $\mathcal{L}_{eff} = c[s\gamma_\mu d]^2$ with coefficient $c \sim 1/\Lambda_{ETC}^2$.

Our key point: in terms of ETC eigenstates, an $s\bar{d}$ in a $\bar{K}^0$ produces a V_1^2 ETC gauge boson, but this cannot directly yield a $d\bar{s}$ in the final-state K^0; the latter is produced by a V_2^1. So this requires either the ETC gauge boson mixing $V_1^2 \longrightarrow V_2^1$ or the mixing of ETC quark eigenstates to produce mass eigenstates. With our reasonably UV-complete ETC model, we have calculated this mixing and shown that it is strongly suppressed because of residual approximate generational symmetries (Appelquist, Piai, and RS, Phys. Rev. D 69, 015002 (2004)).

Contribution from $V_1^2 \longrightarrow V_2^1$:

$$c \sim \frac{1}{\Lambda_1^2}\,{}_2^1\Pi_1^2\,\frac{1}{\Lambda_1^2} \sim \frac{\Lambda_3^2}{\Lambda_1^2}\frac{1}{\Lambda_1^2} \ll \frac{1}{\Lambda_1^2}$$

With above values, $\Lambda_1 \sim 10^3$ TeV, $\Lambda_3 \sim 3$ TeV, the suppression factor is $(\Lambda_3/\Lambda_1)^2 \simeq 10^{-5}$. So rather than the naive result $\Delta m_{K_L K_S}/m_K \sim \Lambda_{QCD}^2/\Lambda_1^2$, this yields the much smaller result

$$\frac{\Delta m_{K_L K_S}}{m_K} \sim \frac{\Lambda_3^2\,\Lambda_{QCD}^2}{\Lambda_1^4} \sim 10^{-18}$$

Hence, the dominant ETC contributions arise from the mixing of ETC eigenstates of quarks to form mass eigenstates.

We have analyzed ETC contributions to a number of FCNC processes and have found that an ETC model with the generational ETC scales given above, namely

$$\Lambda_1 \sim 10^3 \text{ TeV}, \quad \Lambda_2 \sim 10^2 \text{ TeV} , \quad \Lambda_3 \sim \text{ few TeV}$$

can satisfy constraints from experiment (Appelquist, Piai, RS, Phys. Rev. D 69, 015002 (2004); Phys. Lett. B 593, 175 (2004); Phys. Lett. B 595, 442 (2004); Appelquist, Christensen, Piai, RS, Phys. Rev. D 70, 093010 (2004)).

A number of other topics in phenomenology of TC/ETC models are of interest. For example, such models typically contain candidates for dark matter; a number of authors have investigated this.

Constraints from precision electroweak data: $\Delta\rho = \alpha_{em} T$ and S.

$$\frac{\alpha_{em} S}{\sin^2(2\theta_W)} = \frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2}$$

S is sensitive to heavy fermion loop contributions to Z propagator. Naive perturbative estimate:

$$(\Delta S)_{TC,pert.} \simeq \frac{dim(R_{TC})\, N_D}{6\pi}$$

where $dim(R_{TC})$ is the dimension of the TC fermion rep., e.g., $dim(R_{TC}) = N_{TC}$ for fundamental. Using the minimal choice for N_{TC}, namely $N_{TC} = 2$, helps to reduce TC contrib. to S.

Walking reduces $(\Delta S)_{TC}$ (Appelquist and Triantaphyllou, 1992; Sundrum and Hsu, 1993; Ignjatovic, Wijewardhana, and Takeuchi, 1997; Appelquist and Sannino, 1999).

There have been several recent studies of walking reduction of $(\Delta S)_{TC}$ using solutions of Dyson-Schwinger and Bethe-Salpeter equations (Harada, Kurachi and Yamawaki, Prog. Theor. Phys. 115, 765 (2006); Kurachi and RS, Phys. Rev. D 74, 056003 (2006)). Studies using other approaches (Casalbuoni et al., 1995-2008; Chivukula et al., 2002-2008; Hong and Yee 2006; Hirn and Sanz 2006, 2007;, Piai 2007; Fabbrichesi, Piai, Vecchi, 2008; Csaki et al. 2007).

To reduce S, one could also reduce N_D, reconsider one-doublet (1DTC) TC models. An SU(2)$_{TC}$ model with one SU(2)$_L$ doublet of technifermions in the fundamental rep. does not have walking, which is needed not just to reduce S but also to generate adequate fermion masses.

One can retain the desired walking behavior in a TC model with only one SU(2)$_L$ doublet of technifermions by adding SM-singlet technifermions (e.g., Christensen and RS, Phys. Lett. B 632, 92 (2006) and refs. therein). Note that in ETC models using a 1DTC sector, $[G_{ETC}, G_{SM}] \neq 0$; the ETC gauge bosons carry color and charge.

Much work by Sannino, Tuominen, Dietrich, Ryttov, et al. on models that minimize TC contributions to S and can exhibit walking, based on SU(2)$_{TC}$ with two technifermions in the adjoint representation, transforming as one SU(2)$_L$ doublet plus RH SU(2)$_L$ singlets; also studies mapping the boundary of the conformal phase for different representations (and generalization to case of multiple representations (Ryttov...)).

It is also useful to calculate S taking into account not just TC contributions but also ETC effects (Kurachi, RS, Yamawaki, Phys. Rev. D 76, 035003 (2007)). It is found that ETC effects do not increase S significantly.

Currently, several groups are undertaking lattice calculations of S in walking theories. It will be interesting to see what they find and how the results compare with approximate continuum estimates.

In general, the constraint from the S parameter remains a crucial one for TC/ETC theories.

Splitting of t-b Masses

Another challenge for TC/ETC models is to get splitting of m_t and m_b without excessive contributions to ρ. One cannot do this via $\Sigma_{TC,U} > \Sigma_{TC,D}$, because $\Sigma_{TC,U} = \Sigma_{TC,D}$ to good accuracy in TC (and anyway, this would violate custodial symmetry too greatly).

One could consider trying to achieve this splitting using a class of ETC models in which left and right components of up-type quarks and techniquarks transform the same way under SU(5)$_{ETC}$ but the left and right components of down-type quarks and techniquarks transform according to relatively conjugate reps. However, we showed that these are excluded because of excessive FCNC's (Appelquist, Piai, RS, op. cit., Appelquist, Christensen, and Piai, RS, op. cit.). For example, since Q_L and u_R are 5's of SU(5)$_{ETC}$, and d_R is a $\bar{5}$, $\bar{K}^0 \leftrightarrow K^0$ directly via $s_L \bar{d}_L \to V_1^2 \to d_R \bar{s}_R$ without ETC gauge boson mixing. This gives too large $\Delta m_{K_L K_S}$. (See also Christensen and RS, Phys. Rev. D 74, 015004 (2006)).

Explaining $t - b$ splitting while satisfying other constraints remains a challenge for ETC models (perhaps suggesting additional new dynamics such as topcolor).

An important property of models with dynamical EWSB is that running SM fermion masses have power-law decays with momentum scale at momenta above the ETC scale where they are generated. We have investigated whether precision electroweak data are consistent with these power-law decays of SM fermion masses and have shown that they are (Christensen and RS, Phys. Rev. Lett. 94, 241801 (2005)).

The largest effects occur for m_t. Consider, e.g., the t quark contribution to ρ. The conventional (hard, one-loop, 1ℓ) result for this is

$$(\Delta\rho)_{tb,hard,1\ell} = \frac{N_c G_F f_\rho(m_t^2, m_b^2)}{8\pi^2\sqrt{2}} \simeq \frac{3G_f m_t^2}{8\pi^2\sqrt{2}}$$

where $f_\rho(x,y) = x + y - 2xy(x-y)^{-1}\ln(x/y)$.

The power-law decay of m_t above Λ_3 yields

$$(\Delta\rho)_{tb} = (\Delta\rho)_{tb,hard}\left[1 - a_\rho\left(\frac{m_t^2}{\Lambda_3^2}\right)\right]$$

where a_ρ is positive, $\sim O(1)$. The softness of m_t slightly reduces the violation of custodial symmetry; $(m_t/\Lambda_3)^2 = 0.03(1\ \mathrm{TeV}/\Lambda_3)^2$, so fractional reduction of $\Delta\rho_{tb}$ and hence T is quite small, of order $10^{-2} - 10^{-3}$. We find a similarly small change in the (t,b) contribution to S relative to the conventional hard-mass result.

Dynamical Symmetry Breaking of Higher Gauge Symmetries

Question: to what extent can one embed TC, ETC in a theory having higher gauge unification, using dynamical symmetry breaking? This would be desirable in order to explain features not explained by the standard model - (i) unification of quarks and leptons and (ii) charge quantization. We have shown that this is possible (Appelquist and RS, Phys. Rev. Lett. $\mathbf{90}$, 201801 (2003)).

First, consider two extended strong-electroweak groups, G_{LR} and G_{422}

$$G_{LR} = \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$$

in which the SM fermions of each generation transform as

$$(3,2,1)_{1/3,L} \ , \quad (3,1,2)_{1/3,R} \ , \quad (1,2,1)_{-1,L} \ , \quad (1,1,2)_{-1,R}$$

The gauge couplings are defined via the covariant derivative

$$D_\mu = \partial_\mu - ig_3 \mathrm{T}_c \cdot \mathrm{A}_{c,\mu} - ig_{2L} \mathrm{T}_L \cdot \mathrm{A}_{L,\mu} - ig_{2R} \mathrm{T}_R \cdot \mathrm{A}_{R,\mu} - i(g_U/2)(B-L)\mathrm{U}_\mu$$

and the electric charge operator is

$$Q = T_{3L} + T_{3R} + \frac{B-L}{2}$$

where $B =$ baryon no., $L =$ lepton number. Given experimental limits on right-handed charged currents and an associated W_R, and on extra Z''s, SU(2)$_R$ must be broken at a scale Λ_{LR} well above the electroweak scale, and similarly with U(1)$_{B-L}$.

The second extended gauge group is

$$G_{422} = \mathrm{SU}(4)_{PS} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$$

(Pati, Salam, Mohapatra,. ..) with the SM fermions transforming as

$$(4, 2, 1)_L \ , \quad (4, 1, 2)_R$$

G_{422} provides a higher degree of unification since:

- It partially unifies quarks and leptons, e.g., for the first-generation,

$$\begin{pmatrix} u^a & \nu_e \\ d^a & e \end{pmatrix}_{L,R}$$

- It combines U(1)$_{B-L}$ and SU(3)$_c$ (in a maximal subgroup) in SU(4)$_{PS}$ and hence relates g_U and g_3. Denote $T_{PS,15} = (2\sqrt{6})^{-1}\mathrm{diag}(1, 1, 1, -3)$ and set

$U_\mu = A_{PS,15,\mu}$; then $(B-L)/2 = \sqrt{2/3}\, T_{PS,15}$, so, with $g_{PS} = g_3$, we have $g_U^2/g_3^2 = 3/2$.

- It quantizes electric charge:

$$Q = T_{3L} + T_{3R} + \sqrt{2/3}\, T_{PS,15} = T_{3L} + T_{3R} + (1/6)\mathrm{diag}(1,1,1,-3)$$

The quark-lepton unification in the G_{422} theory does not lead to proton decay, but it does lead to the decays such as $K_L \to \mu^\pm e^\mp$ and $K^+ \to \pi^+ \mu^+ e^-$; consistency with experimental upper limits on these decays implies that the scale Λ_{PS} at which SU(4)$_{PS}$ breaks is $\Lambda_{PS} \gtrsim 300$ TeV.

The conventional approach to the gauge symmetry breaking in these models employs elementary Higgs fields, with same ad hoc negative μ^2's and hierarchy problem as in the SM. Instead, we have constructed a dynamical theory, which explains (i) why the $\mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ symmetry of the G_{LR} model breaks to U(1)$_Y$, (ii) why the $\mathrm{SU}(4)_{PS} \times \mathrm{SU}(2)_R$ part of the G_{422} model breaks to $\mathrm{SU}(3)_c \times \mathrm{U}(1)_Y$, and (iii) why the associated breaking scales Λ_{LR} and Λ_{PS} are large wrt. the electroweak scale.

The model with G_{LR} uses the full gauge group

$$G = \mathrm{SU}(5)_{ETC} \times \mathrm{SU}(2)_{HC} \times G_{LR}$$

with SM fermions contained in

$$(5,1,3,2,1)_{1/3,L}\ , \quad (5,1,3,1,2)_{1/3,R}\ , \quad (5,1,1,2,1)_{-1,L}\ , \quad (5,1,1,1,2)_{-1,R}$$

The SM-singlet fermion sector is

$$\mathcal{N}_{i,R}:\ (\bar{5},1,1,1,1)_{0,R}\ , \quad \psi_{ij,R}:\ (\overline{10},1,1,1,1)_{0,R}\ ,$$
$$\zeta_R^{ij,\alpha}:\ (10,2,1,1,1)_{0,R}\ , \quad \omega_{p,R}^{\alpha}\ :\ 2(1,2,1,1,1)_{0,R}$$

As before, the SU(5)$_{ETC}$ model is an asymptotically free, anomaly-free, chiral gauge theory, and SU(2)$_{HC}$ is an asymptotically free (vectorial) gauge interaction.

As the energy decreases from some high value, the SU(5)$_{ETC}$ and SU(2)$_{HC}$ couplings thus increase. We envision that at $E \sim \Lambda_{LR} \gtrsim 10^3$ TeV, $\alpha_{_{ETC}}$ is sufficiently strong to produce condensation in the channel

$$(5,1,1,1,2)_{-1,R} \times (\bar{5},1,1,1,1)_{0,R} \to (1,1,1,1,2)_{-1}$$

with $\Delta C_2 = 24/5$, breaking $\mathrm{SU}(2)_R \times U(1)_{B-L}$ to U(1)$_Y$. The associated condensate is $\langle {n_R^i}^T C \mathcal{N}_{i,R}\rangle$. The n_R^i and $\mathcal{N}_{i,R}$ thus gain dynamical masses $\sim \Lambda_{LR}$.

This condensation generates masses

$$m_{W_R} = \frac{g_{2R}}{2}\Lambda_{LR} \qquad m_{Z'} = \frac{g_{2u}}{2}\Lambda_{LR}$$

where $g_{2u} \equiv \sqrt{g_{2R}^2 + g_U^2}$, for the $W^{\pm}_{R,\mu} = A^{\pm}_{R,\mu}$ gauge bosons and the linear combination $Z'_\mu = (g_{2R}A_{3,R,\mu} - g_U U_\mu)/g_{2u}$. This leaves the orthogonal combination $B_\mu = (g_U A_{3,R,\mu} + g_{2R}U_\mu)/g_{2u}$ as the weak hypercharge U(1)$_Y$ gauge boson. For $E < \Lambda_{LR}$, the effective theory has gauge symmetry $\mathrm{SU}(5)_{ETC} \times \mathrm{SU}(2)_{HC} \times G_{SM}$ and the same fermion content as in our first ETC model. Hence, our analysis for that model can be taken over and applied here.

We obtain similar results for the theory with the theory based on group G_{422}, thereby achieving charge quantization within the context of dynamical symmetry breaking and also showing how the G_{422} symmetry breaks to the SM (with EW symmetry then broken by TC in the usual way).

Prospects for possible higher unification of both TC and SM symmetries have also been studied (Farhi and Susskind, Phys. Phys. Rev. D 20, 3404 (1979); Christensen and RS, PR D 72, 035013 (2005); Gudnason, Ryttov, Sannino, Phys. Rev. D 76, 015005 (2007); Chen and RS, PRD 78, 035002).

A Toy Model with EWSB Predominantly Due to QCD

Early inspiration for TC models arose from the realization that QCD dynamically breaks electroweak symmetry. In the real world, this QCD-induced EWSB at the scale Λ_{QCD} is very small compared to the main EWSB at the scale 250 GeV.

Interesting question: how would the world be different if the QCD-induced EWSB were dominant? A study of models of this type reveals many striking differences with the real world and gives insight into the role of the EWSB scale in nature (C. Quigg and RS, Phys. Rev. D79, 096002 (2009)).

Consider reduced Standard Models (RSM's) in which the real-world 250 GeV-scale EWSB is absent; focus on two types of models:

1. RSM1, with no bare mass terms for the quarks and leptons,
2. RSM2, with bare mass terms for quarks and leptons, explicitly violating EW symmetry, restricted to be sufficiently small that the model serves as a reasonable low-energy effective field theory up to energy scales well above the QCD scale.

The SU(3)$_c$, SU(2)$_L$, and U(1)$_Y$ gauge couplings g_s, g, and g' are taken to have approximately their actual values.

As noted, the would-be Nambu-Goldstone bosons $\pi^\pm$ and π^0 are absorbed to become the longitudinal components of the $W^\pm$ and Z, with masses (marked here with bars to distinguish them from the real-world m_W and m_Z)

$$\bar{m}_W^2 = \frac{g^2 N_D f_\pi^2}{4}\,, \qquad \bar{m}_Z^2 = \frac{(g^2 + g'^2) N_D f_\pi^2}{4}$$

where $N_D = N_g =$ number of quark SU(2)$_L$ doublets. For $N_g = 1$, $m_W \sim 30$ MeV, $m_Z \sim 33$ MeV.

There is a new type of unification of weak and residual strong interactions here, resulting from the absorption of the would-be π's to make the electroweak vector bosons W and Z.

The strength of charged- and neutral-current weak interactions is given by the Fermi coupling

$$\frac{\bar{G}_F}{\sqrt{2}} = \frac{g^2}{8\bar{m}_W^2} = \frac{1}{2f_\pi^2} = \frac{g^2 + g'^2}{8\bar{m}_Z^2}$$

This is much larger than in the real world:

$$\frac{\bar{G}_F}{G_F} = \frac{v^2}{f_\pi^2} \simeq 0.7 \times 10^7$$

where $v = 246$ GeV is the real-world EWSB scale.

For weak decays and charged-current (CC) and neutral-current (NC) cross sections with momentum transfers small compared with $\bar{m}_W$ and $\bar{m}_Z$, the effective strength of weak interactions $\propto G_F^2$, is a factor of $\sim 10^{13}$ larger than in the real world and is much closer to the strength of the residual strong interactions.

For example, if, as is plausible, the masses of the nucleons p and n differ by a few MeV, then the heavier nucleon beta-decays to the lighter with a lifetime $\tau \sim (f_\pi/v)^4 \tau_n \sim 10^{-11}$ sec. where $\tau_n \simeq 0.9 \times 10^3$ s.

The long-range component of the residual strong interactions between color-singlet hadrons in the real world is mediated by pion exchange, with range $\sim 1/m_\pi = 1.4$ fm. Here there are no pions, as such, in the hadron spectrum, but instead there are the very low-mass W and Z. Pion exchange is replaced by the weak CC and NC exchange of the W and Z, with the greater range $1/\bar{m}_{W,Z} \sim 6$ fm. These interactions violate P and C.

Effect on nucleon binding to form nuclei: A simple description of NN binding uses a solution of the Schrödinger equation in a square-well potential. Let the radial size of the square well be a and the depth V_0. The occurrence and number of bound states is determined by the dimensionless parameter

$$\xi = \frac{2\mu V_0 a^2}{\hbar^2 \pi^2}$$

where μ is the reduced mass, i.e., $M_N/2$ for the NN system. When $\xi > O(1)$ a first bound state appears, and as ξ increases, more bound states appear in the spectrum. A figure of merit is $V_0 a^2$. Now

$$\frac{\bar{a}}{a_{rw}} = \frac{(1/\bar{m}_{W,Z})}{(1/m_\pi)} = \frac{2m_\pi}{g f_\pi} \simeq 4.5$$

and since $V_0 \sim$ amplitude for π or $W,\ Z$ exchange, we have

$$\frac{\bar{V}_0}{(V_0)_{rw}} = \frac{g^2/(8\bar{m}_W^2)}{g_{\pi NN}^2/m_\pi^2} = \frac{1/(2f_\pi^2)}{g_{\pi NN}^2/m_\pi^2}$$

so

$$\frac{\bar{\xi}}{\xi_{rw}} = \frac{2m_\pi^4}{g_{\pi NN}^2 g^2 f_\pi^4} \sim \frac{1}{6}$$

Thus, the fact that the coupling g^2 for the $W,\ Z$ exchange in this world is smaller than the coupling $g_{\pi NN}$ for π exchange in the real world is partially cancelled by the greater range, $\bar{a} > a_{rw}$.

Since the p and n masses are mainly due to confinement energy of the quarks and gluons or equivalently to the dynamically generated constituent quark masses, taking the $m_u,\ m_d \longrightarrow 0$ only reduces $m_p,\ m_n$ slightly.

The absence of a Higgs boson means that the perturbatively calculated partial wave amplitudes (PWA's) for $W,\ Z$ scattering exceed unitarity at a scale of few $\times \Lambda_{QCD}$, reflecting the formation of hadronic bound states - ρ, etc. This is understandable, since perturbation theory should not hold in the presence of strong interactions that produce bound states. The full amplitudes including the effect of the resonances do, of course, obey unitarity; the resonances "unitarize" the amplitudes.

For $s >> \bar{m}_W^2,\ \bar{m}_Z^2$, the matrix of $J = 0$ partial wave amplitudes for $2 \longrightarrow 2$ scattering of the normalized states $|W^+W^-\rangle$ and $|ZZ/\sqrt{2}\rangle$ is

$$a_0 = \frac{s}{32\pi N_g f_\pi^2} \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & 0 \end{pmatrix}$$

The larger eigenvalue is $a_{0,max} = s/(16\pi N_g f_\pi^2)$. Imposing the unitarity condition $|a_0| \le 1$ yields the inequality $\sqrt{s} < 4\sqrt{N_g \pi}\, f_\pi$, i.e., $\simeq 640$ MeV for $N_g = 1$, for the perturbatively calculated PWA's to hold. This makes sense, since the nonperturbative bound state, ρ, with mass comparable to its real-world value, 775 MeV, is just above this.

The EWSB by the $\langle \bar{q}q \rangle$ condensate does not directly give masses to the SM fermions. These fermion masses are dependent on the UV completion of the theory. If one assumes nothing further, then there is an infrared pathology due to the presence of the massless charged unconfined electron. Among other things, this leads to the collapse of the vacuum into a plasma due to the production of an avalanche of e^+e^- pairs by arbitrarily soft photons and arbitrarily weak electric fields.

We have also studied another toy model, RSM2, with bare SM fermion mass terms. This has the advantage of being free of any IR pathology and still allowing a study of the properties of a model in which EWSB can be dominantly due to the QCD condensate, for small enough m_f.

Conclusions

Dynamical electroweak symmetry breaking is an interesting and well-motivated possibility. This approach is very ambitious and encounters a number of challenges. So far, studies have shown

- how a new gauge interaction that becomes strongly coupled on the TeV scale naturally produces EWSB, W and Z masses

- how an associated large but slowly running gauge coupling can result from an approximate IR fixed point, enhancing fermion mass generation and leading to reduction in TC modifications of precision electroweak quantities

- how generations can arise, by sequential breaking of ETC symmetry

- how the EWSB can be communicated to the fermions and hence how quark and charged lepton masses can arise, and also how one can get light neutrinos

- how the constraints from FCNC were not as severe as had previously been thought, because of approximate residual generational symmetries

- how to construct theories with higher gauge symmetries that yield charge quantization, in which all symmetry breaking is dynamical.

Recently, there has been considerable progress from lattice simulations in determining how the boundary of the chirally symmetric phase of a non-abelian gauge theory depends on the number and representation content of the fermions. The knowledge being gained is of fundamental field-theoretic importance, as well as being crucial for models of dynamical EWSB.

This is a very exciting time, when we can anticipate that the LHC will soon tell us whether dynamical EWSB is realized in nature.