In this document, we provide additional information about the generators in the different representations of the group $\Delta(384)$ and the Clebsch Gordan coefficients for the products $\mathbf{r_i} \times \mathbf{r_j}$ in $\Delta(384)$.

1 Generators in Representations of $\Delta(384)$

The group $\Delta(384)$ has 24 irreducible representations: two have dimension one, one dimension two, fourteen have dimension three and seven are of dimension six. The form of the generators a, b, c and d of $\Delta(384)$ in the one-, two- and three-dimensional representations can be found in [1] and some also in [2]. As regards the six-dimensional representations that are called $\mathbf{6_i}$ in [2] they can be matched to those used in [1]. In the latter work the six-dimensional representations are characterized by two indices k and l that can take values between 0 and 7. Note that these six-dimensional representations are not irreducible for all combinations of indices k and l, see for details [1]. In [2] we only consider seven inequivalent irreducible six-dimensional representations. They can be obtained from the following combinations of indices k and l: $(k,l) = (1,1)$ leads to $\mathbf{6_1}$, $(k,l) = (1,2)$ to $\mathbf{6_2}$, $(k,l) = (1,3)$ to $\mathbf{6_3}$, $(k,l) = (1,4)$ to $\mathbf{6_4}$, $(k,l) = (1,5)$ to $\mathbf{6_5}$, $(k,l) = (2,2)$ to $\mathbf{6_6}$ and $(k,l) = (2,3)$ to $\mathbf{6_7}$. As can be checked, the representations $\mathbf{6_1}$, $\mathbf{6_6}$ and $\mathbf{6_7}$ are real, while $\mathbf{6_2}$ and $\mathbf{6_5}$ as well as $\mathbf{6_3}$ and $\mathbf{6_4}$ are complex conjugated to each other. Furthermore, the six-dimensional representation $\mathbf{6_6}$ is unfaithful with respect to $\Delta(384)$. The form of the generators a and b is the same in all representations $\mathbf{6_i}$ and reads

$$a(\mathbf{6_i}) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad b(\mathbf{6_i}) = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}, \tag{1}$$

while the one of c and d depends on the index of the six-dimensional representation:

$$c(\mathbf{6_1}) = \begin{pmatrix} \omega_8 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_8^6 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_8^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_8^7 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_8^7 \end{pmatrix} \quad \text{and} \quad d(\mathbf{6_1}) = \begin{pmatrix} \omega_8^6 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_8^7 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_8^7 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_8^2 \end{pmatrix},$$

$$c(\mathbf{6_2}) = \begin{pmatrix} \omega_8^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_8^5 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_8^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_8^6 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_8^7 \end{pmatrix} \quad \text{and} \quad d(\mathbf{6_2}) = \begin{pmatrix} \omega_8^5 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_8^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_8^6 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_8^7 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_8^3 \end{pmatrix},$$

$$c(\mathbf{6_3}) = \begin{pmatrix} \omega_8^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_8^4 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_8^4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_8^5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_8^7 \end{pmatrix} \quad \text{and} \quad d(\mathbf{6_3}) = \begin{pmatrix} \omega_8^4 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_8^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_8^5 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_8^7 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_8^4 \end{pmatrix},$$

$$c(\mathbf{6_4}) = \begin{pmatrix} \omega_8^4 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_8^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_8^5 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_8^4 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_8^7 \end{pmatrix} \text{ and } d(\mathbf{6_4}) = \begin{pmatrix} \omega_8^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_8^4 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_8^4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_8^7 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_8^5 \end{pmatrix},$$

$$c(\mathbf{6_5}) = \begin{pmatrix} \omega_8^5 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_8^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_8^6 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_8^3 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_8^7 \end{pmatrix} \text{ and } d(\mathbf{6_5}) = \begin{pmatrix} \omega_8^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_8^5 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_8^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_8^7 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_8^6 \end{pmatrix},$$

$$c(\mathbf{6_6}) = \begin{pmatrix} \omega_8^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_8^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_8^4 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_8^4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_8^6 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_8^6 \end{pmatrix} \text{ and } d(\mathbf{6_6}) = \begin{pmatrix} \omega_8^4 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_8^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_8^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_8^6 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_8^6 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_8^4 \end{pmatrix},$$

$$c(\mathbf{6_7}) = \begin{pmatrix} \omega_8^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_8^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_8^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_8^5 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_8^5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_8^6 \end{pmatrix} \text{ and } d(\mathbf{6_7}) = \begin{pmatrix} \omega_8^3 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega_8^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega_8^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_8^5 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_8^6 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_8^5 \end{pmatrix} \quad (2)$$

with $\omega_8 = e^{2\pi i/8}$.

2 Clebsch Gordan Coefficients for $\mathbf{r_i} \times \mathbf{r_j}$ in $\mathbf{\Delta(384)}$

The products of the trivial representation $\mathbf{1}$ with another representation $\mathbf{r}$ of $\Delta(384)$ are straightforward and read for $y \sim \mathbf{1}$ and $z_i \sim \mathbf{r}$

$$z \sim \mathbf{1} : \quad y\,z \sim \mathbf{1} \quad , \quad z \sim \mathbf{1^-} : \quad y\,z \sim \mathbf{1^-} \quad , \quad z \sim \mathbf{2} : \quad \begin{pmatrix} y\,z_1 \\ y\,z_2 \end{pmatrix} \sim \mathbf{2},$$
$$z \sim \mathbf{3_i} : \quad \begin{pmatrix} y\,z_1 \\ y\,z_2 \\ y\,z_3 \end{pmatrix} \sim \mathbf{3_i} \quad , \quad z \sim \mathbf{3_i^-} : \quad \begin{pmatrix} y\,z_1 \\ y\,z_2 \\ y\,z_3 \end{pmatrix} \sim \mathbf{3_i^-} \quad , \quad z \sim \mathbf{6_i} : \quad \begin{pmatrix} y\,z_1 \\ y\,z_2 \\ y\,z_3 \\ y\,z_4 \\ y\,z_5 \\ y\,z_6 \end{pmatrix} \sim \mathbf{6_i}\,. \quad (3)$$

The products of the non-trivial one-dimensional representation $\mathbf{1^-}$ with another representation $\mathbf{r}$ have a similar form and read for $y \sim \mathbf{1^-}$ and $z_i \sim \mathbf{r}$

$$z \sim \mathbf{1} : \quad y\,z \sim \mathbf{1^-} \quad , \quad z \sim \mathbf{1^-} : \quad y\,z \sim \mathbf{1} \quad , \quad z \sim \mathbf{2} : \quad \begin{pmatrix} y\,z_1 \\ -y\,z_2 \end{pmatrix} \sim \mathbf{2},$$
$$z \sim \mathbf{3_i} : \quad \begin{pmatrix} y\,z_1 \\ y\,z_2 \\ y\,z_3 \end{pmatrix} \sim \mathbf{3_i^-} \quad , \quad z \sim \mathbf{3_i^-} : \quad \begin{pmatrix} y\,z_1 \\ y\,z_2 \\ y\,z_3 \end{pmatrix} \sim \mathbf{3_i} \quad , \quad z \sim \mathbf{6_i} : \quad \begin{pmatrix} y\,z_1 \\ y\,z_2 \\ y\,z_3 \\ -y\,z_4 \\ -y\,z_5 \\ -y\,z_6 \end{pmatrix} \sim \mathbf{6_i}\,. \quad (4)$$

Combining the two-dimensional representation $\mathbf{2}$ with another representation $\mathbf{r}$ of dimension larger than one leads to the following covariants for $y_i \sim \mathbf{2}$ and $z_j \sim \mathbf{r}$

$$
\begin{array}{llll}
z \sim \mathbf{2}: & y_1 z_2 + y_2 z_1 \sim \mathbf{1} \quad , & y_1 z_2 - y_2 z_1 \sim \mathbf{1}^- \quad , & \begin{pmatrix} y_2 z_2 \\ y_1 z_1 \end{pmatrix} \sim \mathbf{2}\,, \\
z \sim \mathbf{3_i}: & \begin{pmatrix} y_1 z_1 + \omega^2 y_2 z_1 \\ \omega y_1 z_2 + \omega y_2 z_2 \\ \omega^2 y_1 z_3 + y_2 z_3 \end{pmatrix} \sim \mathbf{3_i} \quad , & \begin{pmatrix} y_1 z_1 - \omega^2 y_2 z_1 \\ \omega y_1 z_2 - \omega y_2 z_2 \\ \omega^2 y_1 z_3 - y_2 z_3 \end{pmatrix} \sim \mathbf{3_i^-}\,, & \\
z \sim \mathbf{3_i^-}: & \begin{pmatrix} y_1 z_1 - \omega^2 y_2 z_1 \\ \omega y_1 z_2 - \omega y_2 z_2 \\ \omega^2 y_1 z_3 - y_2 z_3 \end{pmatrix} \sim \mathbf{3_i} \quad , & \begin{pmatrix} y_1 z_1 + \omega^2 y_2 z_1 \\ \omega y_1 z_2 + \omega y_2 z_2 \\ \omega^2 y_1 z_3 + y_2 z_3 \end{pmatrix} \sim \mathbf{3_i^-}\,, & \\
z \sim \mathbf{6_i}: & \begin{pmatrix} y_1 z_1 - y_2 z_1 \\ \omega y_1 z_2 - \omega^2 y_2 z_2 \\ \omega^2 y_1 z_3 - \omega y_2 z_3 \\ -y_1 z_4 + y_2 z_4 \\ -\omega^2 y_1 z_5 + \omega y_2 z_5 \\ -\omega y_1 z_6 + \omega^2 y_2 z_6 \end{pmatrix} \sim (\mathbf{6_i})_1 \quad , & \begin{pmatrix} y_1 z_1 + y_2 z_1 \\ \omega y_1 z_2 + \omega^2 y_2 z_2 \\ \omega^2 y_1 z_3 + \omega y_2 z_3 \\ y_1 z_4 + y_2 z_4 \\ \omega^2 y_1 z_5 + \omega y_2 z_5 \\ \omega y_1 z_6 + \omega^2 y_2 z_6 \end{pmatrix} \sim (\mathbf{6_i})_2\,. &
\end{array}
\tag{5}
$$

with $\omega = e^{2\pi i/3}$ and $(\mathbf{6_i})_1$ and $(\mathbf{6_i})_2$ denoting two independent covariants in the product $\mathbf{2} \times \mathbf{6_i}$ that both transform as six-dimensional representation $\mathbf{6_i}$.

As regards the products of two three-dimensional representations, we first discuss the products $\mathbf{3_i} \times \mathbf{3_i}$ and $\mathbf{3_i^-} \times \mathbf{3_i^-}$ for $\mathrm{i} \neq 4$. These products contain three covariants, the three-dimensional representations $\mathbf{3_{2\,i \bmod 8}}$, $\mathbf{3_{8-i}}$ and $\mathbf{3_{8-i}^-}$, that are of the form

$$
\begin{pmatrix} y_1 z_1 \\ y_2 z_2 \\ y_3 z_3 \end{pmatrix} \sim \mathbf{3_{2\,i \bmod 8}}\,, \quad \begin{pmatrix} y_2 z_3 + y_3 z_2 \\ y_1 z_3 + y_3 z_1 \\ y_1 z_2 + y_2 z_1 \end{pmatrix} \sim \mathbf{3_{8-i}} \text{ and } \begin{pmatrix} y_2 z_3 - y_3 z_2 \\ -y_1 z_3 + y_3 z_1 \\ y_1 z_2 - y_2 z_1 \end{pmatrix} \sim \mathbf{3_{8-i}^-}\,. \tag{6}
$$

For the product $\mathbf{3_i} \times \mathbf{3_i^-}$ with $\mathrm{i} \neq 4$ we find similarly as covariants

$$
\begin{pmatrix} y_1 z_1 \\ y_2 z_2 \\ y_3 z_3 \end{pmatrix} \sim \mathbf{3^-_{2\,i \bmod 8}}\,, \quad \begin{pmatrix} y_2 z_3 - y_3 z_2 \\ -y_1 z_3 + y_3 z_1 \\ y_1 z_2 - y_2 z_1 \end{pmatrix} \sim \mathbf{3_{8-i}} \text{ and } \begin{pmatrix} y_2 z_3 + y_3 z_2 \\ y_1 z_3 + y_3 z_1 \\ y_1 z_2 + y_2 z_1 \end{pmatrix} \sim \mathbf{3_{8-i}^-} \tag{7}
$$

with $y_j \sim \mathbf{3_i}$ and $z_k \sim \mathbf{3_i^-}$. For $\mathbf{3_4} \times \mathbf{3_4}$ and $\mathbf{3_4^-} \times \mathbf{3_4^-}$ we find instead

$$
\begin{array}{c}
y_1 z_1 + y_2 z_2 + y_3 z_3 \sim \mathbf{1}\,, \quad \begin{pmatrix} y_1 z_1 + \omega^2 y_2 z_2 + \omega y_3 z_3 \\ \omega y_1 z_1 + \omega^2 y_2 z_2 + y_3 z_3 \end{pmatrix} \sim \mathbf{2}\,, \\
\begin{pmatrix} y_2 z_3 + y_3 z_2 \\ y_1 z_3 + y_3 z_1 \\ y_1 z_2 + y_2 z_1 \end{pmatrix} \sim \mathbf{3_4} \text{ and } \begin{pmatrix} y_2 z_3 - y_3 z_2 \\ -y_1 z_3 + y_3 z_1 \\ y_1 z_2 - y_2 z_1 \end{pmatrix} \sim \mathbf{3_4^-}\,.
\end{array}
\tag{8}
$$

The covariants of the product $\mathbf{3_4} \times \mathbf{3_4^-}$ are of similar form

$$
\begin{array}{c}
y_1 z_1 + y_2 z_2 + y_3 z_3 \sim \mathbf{1}^-\,, \quad \begin{pmatrix} y_1 z_1 + \omega^2 y_2 z_2 + \omega y_3 z_3 \\ -\left(\omega y_1 z_1 + \omega^2 y_2 z_2 + y_3 z_3\right) \end{pmatrix} \sim \mathbf{2}\,, \\
\begin{pmatrix} y_2 z_3 - y_3 z_2 \\ -y_1 z_3 + y_3 z_1 \\ y_1 z_2 - y_2 z_1 \end{pmatrix} \sim \mathbf{3_4} \text{ and } \begin{pmatrix} y_2 z_3 + y_3 z_2 \\ y_1 z_3 + y_3 z_1 \\ y_1 z_2 + y_2 z_1 \end{pmatrix} \sim \mathbf{3_4^-}
\end{array}
\tag{9}
$$

for $y_i \sim \mathbf{3_4}$ and $z_j \sim \mathbf{3_4^-}$. The products $\mathbf{3_i} \times \mathbf{3_j}$ and $\mathbf{3_i^-} \times \mathbf{3_j^-}$ for $\mathrm{i} < \mathrm{j}$ and $\mathrm{i} + \mathrm{j} \neq 0 \bmod 8$ contain a three-dimensional and a six-dimensional representation. For $y_a \sim \mathbf{3_i^{(-)}}$ and $z_b \sim \mathbf{3_j^{(-)}}$ they read

$$
\begin{pmatrix} y_1 z_1 \\ y_2 z_2 \\ y_3 z_3 \end{pmatrix} \sim \mathbf{3_{(i+j) \bmod 8}} \text{ and one of the following covariants } (\mathbf{6_k})_1\,,\ (\mathbf{6_k})_2 \text{ or } (\mathbf{6_k})_3 \sim \mathbf{6_k} \tag{10}
$$

with the index k being specified in table 1 and the forms of the six-dimensional representation $(\mathbf{6_k})_\ell$, shown in Eq. (14). Similarly, the covariants of the product $\mathbf{3_i} \times \mathbf{3_j^-}$ for $\mathrm{i} < \mathrm{j}$ and $\mathrm{i} + \mathrm{j} \neq 0 \bmod 8$ read for $y_a \sim \mathbf{3_i}$ and $z_b \sim \mathbf{3_j^-}$

$$\begin{pmatrix} y_1 \, z_1 \\ y_2 \, z_2 \\ y_3 \, z_3 \end{pmatrix} \sim \mathbf{3^-_{(i+j)\ mod\, 8}} \quad \text{and one of the following covariants } (\mathbf{6_k})_{1,-} \,,\ (\mathbf{6_k})_{2,-} \text{ or } (\mathbf{6_k})_{3,-} \sim \mathbf{6_k} \tag{11}$$

with the index k being specified in table 1 and the forms of the six-dimensional representation $(\mathbf{6_k})_{\ell,-}$, shown in Eq. (15). For $\mathrm{i} < \mathrm{j}$ and $\mathrm{i} + \mathrm{j} = 0 \bmod 8$ the products $\mathbf{3_i} \times \mathbf{3_j}$ and $\mathbf{3_i^-} \times \mathbf{3_j^-}$ decompose into the covariants $\mathbf{1}$, $\mathbf{2}$ and a six-dimensional representation $\mathbf{6_k}$

$$y_1 \, z_1 + y_2 \, z_2 + y_3 \, z_3 \sim \mathbf{1} \,, \quad \begin{pmatrix} y_1 \, z_1 + \omega^2 \, y_2 \, z_2 + \omega \, y_3 \, z_3 \\ \omega \, y_1 \, z_1 + \omega^2 \, y_2 \, z_2 + y_3 \, z_3 \end{pmatrix} \sim \mathbf{2} \,,$$
$$\text{and one of the following covariants } (\mathbf{6_k})_1 \,,\ (\mathbf{6_k})_2 \text{ or } (\mathbf{6_k})_3 \sim \mathbf{6_k} \tag{12}$$

with $y_a \sim \mathbf{3_i^{(-)}}$ and $z_b \sim \mathbf{3_j^{(-)}}$ as well as for $\mathbf{3_i} \times \mathbf{3_j^-}$ with $\mathrm{i} < \mathrm{j}$ and $\mathrm{i} + \mathrm{j} = 0 \bmod 8$

$$y_1 \, z_1 + y_2 \, z_2 + y_3 \, z_3 \sim \mathbf{1^-} \,, \quad \begin{pmatrix} y_1 \, z_1 + \omega^2 \, y_2 \, z_2 + \omega \, y_3 \, z_3 \\ -\left(\omega \, y_1 \, z_1 + \omega^2 \, y_2 \, z_2 + y_3 \, z_3\right) \end{pmatrix} \sim \mathbf{2} \,,$$
$$\text{and one of the following covariants } (\mathbf{6_k})_{1,-} \,,\ (\mathbf{6_k})_{2,-} \text{ or } (\mathbf{6_k})_{3,-} \sim \mathbf{6_k} \tag{13}$$

with $y_a \sim \mathbf{3_i}$ and $z_b \sim \mathbf{3_j^-}$.

i	1	1	1	1	1	1	2	2	2	2	2	3	3	3	3	4	4	4	5	5	6
j	2	3	4	5	6	7	3	4	5	6	7	4	5	6	7	5	6	7	6	7	7
k	1	5	4	3	2	1	2	6	7	6	5	3	7	7	4	4	6	3	5	2	1
ℓ	1	2	2	2	2	2	1	1	2	2	3	1	1	3	3	1	3	3	1	3	3

Table 1: Information on the index k and the form $(\mathbf{6_k})_\ell$ and $(\mathbf{6_k})_{\ell,-}$ of the six-dimensional representation contained in the products $\mathbf{3_i} \times \mathbf{3_j}$, $\mathbf{3_i^-} \times \mathbf{3_j^-}$ and $\mathbf{3_i} \times \mathbf{3_j^-}$, respectively.

The form of the covariants, transforming as six-dimensional representation $\mathbf{6_k}$, is one of the following

$$(\mathbf{6_k})_1 \ : \ \begin{pmatrix} y_1 \, z_3 \\ y_2 \, z_1 \\ y_3 \, z_2 \\ y_3 \, z_1 \\ y_2 \, z_3 \\ y_1 \, z_2 \end{pmatrix} \,, \quad (\mathbf{6_k})_2 \ : \ \begin{pmatrix} y_3 \, z_2 \\ y_1 \, z_3 \\ y_2 \, z_1 \\ y_1 \, z_2 \\ y_3 \, z_1 \\ y_2 \, z_3 \end{pmatrix} \,, \quad (\mathbf{6_k})_3 \ : \ \begin{pmatrix} y_2 \, z_1 \\ y_3 \, z_2 \\ y_1 \, z_3 \\ y_2 \, z_3 \\ y_1 \, z_2 \\ y_3 \, z_1 \end{pmatrix} \,, \tag{14}$$

and

$$(\mathbf{6_k})_{1,-} \ : \ \begin{pmatrix} y_1 \, z_3 \\ y_2 \, z_1 \\ y_3 \, z_2 \\ -y_3 \, z_1 \\ -y_2 \, z_3 \\ -y_1 \, z_2 \end{pmatrix} \,, \quad (\mathbf{6_k})_{2,-} \ : \ \begin{pmatrix} y_3 \, z_2 \\ y_1 \, z_3 \\ y_2 \, z_1 \\ -y_1 \, z_2 \\ -y_3 \, z_1 \\ -y_2 \, z_3 \end{pmatrix} \,, \quad (\mathbf{6_k})_{3,-} \ : \ \begin{pmatrix} y_2 \, z_1 \\ y_3 \, z_2 \\ y_1 \, z_3 \\ -y_2 \, z_3 \\ -y_1 \, z_2 \\ -y_3 \, z_1 \end{pmatrix} \,. \tag{15}$$

The products $\mathbf{3_i} \times \mathbf{6_j}$ and $\mathbf{3_i^-} \times \mathbf{6_j}$ for the different indices i and j can be classified according to the representations they contain. For $\mathrm{i} \neq 4$ and a single j the product $\mathbf{3_i} \times \mathbf{6_j}$ and $\mathbf{3_i^-} \times \mathbf{6_j}$, respectively, decomposes into four three-dimensional representations $\mathbf{3_i}$, $\mathbf{3_i^-}$, $\mathbf{3_{(8-2\,i)\ mod\,8}}$ and $\mathbf{3^-_{(8-2\,i)\ mod\,8}}$ and one six-dimensional one $\mathbf{6_k}$. The actual form of the covariants depends on the indices i and j and is listed in table 2 and Eqs. (16) and (17).

i	1	2	3	5	6	7
j	1	6	7	7	6	1
$\mathbf{3_i}$	$\mathrm{Cov}(\mathbf{3},1)$	$\mathrm{Cov}(\mathbf{3},1)$	$\mathrm{Cov}(\mathbf{3},4)$	$\mathrm{Cov}(\mathbf{3},5)$	$\mathrm{Cov}(\mathbf{3},6)$	$\mathrm{Cov}(\mathbf{3},6)$
$\mathbf{3_i^-}$	$\mathrm{Cov}(\mathbf{3},1,-)$	$\mathrm{Cov}(\mathbf{3},1,-)$	$\mathrm{Cov}(\mathbf{3},4,-)$	$\mathrm{Cov}(\mathbf{3},5,-)$	$\mathrm{Cov}(\mathbf{3},6,-)$	$\mathrm{Cov}(\mathbf{3},6,-)$
$\mathbf{3}_{(\mathbf{8-2\,i})\ \mathrm{mod}\,\mathbf{8}}$	$\mathrm{Cov}(\mathbf{3},2)$	$\mathrm{Cov}(\mathbf{3},2)$	$\mathrm{Cov}(\mathbf{3},1)$	$\mathrm{Cov}(\mathbf{3},3)$	$\mathrm{Cov}(\mathbf{3},5)$	$\mathrm{Cov}(\mathbf{3},5)$
$\mathbf{3}^-_{(\mathbf{8-2\,i})\ \mathrm{mod}\,\mathbf{8}}$	$\mathrm{Cov}(\mathbf{3},2,-)$	$\mathrm{Cov}(\mathbf{3},2,-)$	$\mathrm{Cov}(\mathbf{3},1,-)$	$\mathrm{Cov}(\mathbf{3},3,-)$	$\mathrm{Cov}(\mathbf{3},5,-)$	$\mathrm{Cov}(\mathbf{3},5,-)$
k	5	6	5	2	6	2
$\mathbf{6_k}$	$\mathrm{Cov}(\mathbf{6},1)$	$\mathrm{Cov}(\mathbf{6},1)$	$\mathrm{Cov}(\mathbf{6},2)$	$\mathrm{Cov}(\mathbf{6},3)$	$\mathrm{Cov}(\mathbf{6},4)$	$\mathrm{Cov}(\mathbf{6},5)$

Table 2: Information on the form of the covariants $\mathrm{Cov}(\mathbf{3},f)$ and $\mathrm{Cov}(\mathbf{3},f,-)$, transforming as three-dimensional representations, and the index k and the form $\mathrm{Cov}(\mathbf{6},f)$ of the six-dimensional representation contained in the products $\mathbf{3_i}\times\mathbf{6_j}$. For the products $\mathbf{3_i^-}\times\mathbf{6_j}$ exchange the covariants $\mathrm{Cov}(\mathbf{3},f)$ and $\mathrm{Cov}(\mathbf{3},f,-)$ with each other and use $\mathrm{Cov}(\mathbf{6},f,-)$ instead of $\mathrm{Cov}(\mathbf{6},f)$.

For $\mathrm{i}\neq 4$ and four indices j the product $\mathbf{3_i}\times\mathbf{6_j}$ and $\mathbf{3_i^-}\times\mathbf{6_j}$, respectively, decomposes into two three-dimensional representations $\mathbf{3_k}$ and $\mathbf{3_k^-}$ and two six-dimensional ones $\mathbf{6_l}$ and $\mathbf{6_m}$. The actual form of the covariants depends on the indices i and j and is listed in table 3 and Eqs. (16) and (17).

i	1	1	1	1	2	2	2	2
j	2	3	4	5	1	2	5	7
k	5	4	3	2	7	1	5	3
$\mathbf{3_k}$	$\mathrm{Cov}(\mathbf{3},2)$	$\mathrm{Cov}(\mathbf{3},2)$	$\mathrm{Cov}(\mathbf{3},2)$	$\mathrm{Cov}(\mathbf{3},2)$	$\mathrm{Cov}(\mathbf{3},3)$	$\mathrm{Cov}(\mathbf{3},1)$	$\mathrm{Cov}(\mathbf{3},4)$	$\mathrm{Cov}(\mathbf{3},2)$
$\mathbf{3_k^-}$	$\mathrm{Cov}(\mathbf{3},2,-)$	$\mathrm{Cov}(\mathbf{3},2,-)$	$\mathrm{Cov}(\mathbf{3},2,-)$	$\mathrm{Cov}(\mathbf{3},2,-)$	$\mathrm{Cov}(\mathbf{3},3,-)$	$\mathrm{Cov}(\mathbf{3},1,-)$	$\mathrm{Cov}(\mathbf{3},4,-)$	$\mathrm{Cov}(\mathbf{3},2,-)$
l	1	2	3	2	2	3	1	4
m	6	7	6	4	4	7	3	5
$\mathbf{6_l}$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},7)$	$\mathrm{Cov}(\mathbf{6},2)$	$\mathrm{Cov}(\mathbf{6},2)$	$\mathrm{Cov}(\mathbf{6},8)$	$\mathrm{Cov}(\mathbf{6},7)$
$\mathbf{6_m}$	$\mathrm{Cov}(\mathbf{6},6)$	$\mathrm{Cov}(\mathbf{6},6)$	$\mathrm{Cov}(\mathbf{6},7)$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},1)$	$\mathrm{Cov}(\mathbf{6},1)$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},9)$

i	3	3	3	3	5	5	5	5
j	2	3	4	5	2	3	4	5
k	7	1	4	6	2	4	7	1
$\mathbf{3_k}$	$\mathrm{Cov}(\mathbf{3},3)$	$\mathrm{Cov}(\mathbf{3},1)$	$\mathrm{Cov}(\mathbf{3},4)$	$\mathrm{Cov}(\mathbf{3},5)$	$\mathrm{Cov}(\mathbf{3},4)$	$\mathrm{Cov}(\mathbf{3},5)$	$\mathrm{Cov}(\mathbf{3},3)$	$\mathrm{Cov}(\mathbf{3},1)$
$\mathbf{3_k^-}$	$\mathrm{Cov}(\mathbf{3},3,-)$	$\mathrm{Cov}(\mathbf{3},1,-)$	$\mathrm{Cov}(\mathbf{3},4,-)$	$\mathrm{Cov}(\mathbf{3},5,-)$	$\mathrm{Cov}(\mathbf{3},4,-)$	$\mathrm{Cov}(\mathbf{3},5,-)$	$\mathrm{Cov}(\mathbf{3},3,-)$	$\mathrm{Cov}(\mathbf{3},1,-)$
l	6	4	1	2	4	1	3	6
m	7	6	2	3	5	5	6	7
$\mathbf{6_l}$	$\mathrm{Cov}(\mathbf{6},1)$	$\mathrm{Cov}(\mathbf{6},7)$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},8)$	$\mathrm{Cov}(\mathbf{6},4)$	$\mathrm{Cov}(\mathbf{6},11)$	$\mathrm{Cov}(\mathbf{6},11)$
$\mathbf{6_m}$	$\mathrm{Cov}(\mathbf{6},2)$	$\mathrm{Cov}(\mathbf{6},8)$	$\mathrm{Cov}(\mathbf{6},8)$	$\mathrm{Cov}(\mathbf{6},10)$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},10)$	$\mathrm{Cov}(\mathbf{6},3)$	$\mathrm{Cov}(\mathbf{6},3)$

i	6	6	6	6	7	7	7	7
j	1	2	5	7	2	3	4	5
k	1	3	7	5	6	5	4	3
$\mathbf{3_k}$	$\mathrm{Cov}(\mathbf{3},4)$	$\mathrm{Cov}(\mathbf{3},5)$	$\mathrm{Cov}(\mathbf{3},3)$	$\mathrm{Cov}(\mathbf{3},6)$	$\mathrm{Cov}(\mathbf{3},6)$	$\mathrm{Cov}(\mathbf{3},6)$	$\mathrm{Cov}(\mathbf{3},6)$	$\mathrm{Cov}(\mathbf{3},6)$
$\mathbf{3_k^-}$	$\mathrm{Cov}(\mathbf{3},4,-)$	$\mathrm{Cov}(\mathbf{3},5,-)$	$\mathrm{Cov}(\mathbf{3},3,-)$	$\mathrm{Cov}(\mathbf{3},6,-)$	$\mathrm{Cov}(\mathbf{3},6,-)$	$\mathrm{Cov}(\mathbf{3},6,-)$	$\mathrm{Cov}(\mathbf{3},6,-)$	$\mathrm{Cov}(\mathbf{3},6,-)$
l	3	1	4	2	3	4	5	1
m	5	4	7	3	5	6	7	6
$\mathbf{6_l}$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},3)$	$\mathrm{Cov}(\mathbf{6},3)$	$\mathrm{Cov}(\mathbf{6},4)$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},4)$
$\mathbf{6_m}$	$\mathrm{Cov}(\mathbf{6},8)$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},12)$	$\mathrm{Cov}(\mathbf{6},11)$	$\mathrm{Cov}(\mathbf{6},11)$	$\mathrm{Cov}(\mathbf{6},13)$	$\mathrm{Cov}(\mathbf{6},13)$	$\mathrm{Cov}(\mathbf{6},12)$

Table 3: Information on the index k and the form of the covariants $\mathrm{Cov}(\mathbf{3},f)$ and $\mathrm{Cov}(\mathbf{3},f,-)$, transforming as three-dimensional representations, and the indices l and m and the form $\mathrm{Cov}(\mathbf{6},f)$ of the six-dimensional representations, contained in the products $\mathbf{3_i}\times\mathbf{6_j}$. For the products $\mathbf{3_i^-}\times\mathbf{6_j}$ exchange the covariants $\mathrm{Cov}(\mathbf{3},f)$ and $\mathrm{Cov}(\mathbf{3},f,-)$ with each other and use $\mathrm{Cov}(\mathbf{6},f,-)$ instead of $\mathrm{Cov}(\mathbf{6},f)$.

For i $\neq 4$ and two indices j the products $\mathbf{3_i} \times \mathbf{6_j}$ and $\mathbf{3_i^-} \times \mathbf{6_j}$ decompose into three different six-dimensional representations $\mathbf{6_k}$, $\mathbf{6_l}$ and $\mathbf{6_m}$. Again, the indices k, l and m and the explicit form of the covariants can be found in table 4 and Eq. (17).

i	1	1	2	2	3	3
j	6	7	3	4	1	6
k	3	4	1	2	1	1
l	5	6	4	3	3	4
m	7	7	7	5	6	5
$\mathbf{6_k}$	$\mathrm{Cov}(\mathbf{6},13)$	$\mathrm{Cov}(\mathbf{6},13)$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},4)$	$\mathrm{Cov}(\mathbf{6},14)$
$\mathbf{6_l}$	$\mathrm{Cov}(\mathbf{6},9)$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},2)$	$\mathrm{Cov}(\mathbf{6},7)$	$\mathrm{Cov}(\mathbf{6},1)$	$\mathrm{Cov}(\mathbf{6},2)$
$\mathbf{6_m}$	$\mathrm{Cov}(\mathbf{6},1)$	$\mathrm{Cov}(\mathbf{6},7)$	$\mathrm{Cov}(\mathbf{6},7)$	$\mathrm{Cov}(\mathbf{6},2)$	$\mathrm{Cov}(\mathbf{6},2)$	$\mathrm{Cov}(\mathbf{6},7)$

i	5	5	6	6	7	7
j	1	6	3	4	6	7
k	1	1	2	1	2	3
l	4	2	4	3	4	6
m	6	3	5	7	7	7
$\mathbf{6_k}$	$\mathrm{Cov}(\mathbf{6},1)$	$\mathrm{Cov}(\mathbf{6},3)$	$\mathrm{Cov}(\mathbf{6},3)$	$\mathrm{Cov}(\mathbf{6},4)$	$\mathrm{Cov}(\mathbf{6},6)$	$\mathrm{Cov}(\mathbf{6},6)$
$\mathbf{6_l}$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},4)$	$\mathrm{Cov}(\mathbf{6},11)$	$\mathrm{Cov}(\mathbf{6},3)$	$\mathrm{Cov}(\mathbf{6},11)$	$\mathrm{Cov}(\mathbf{6},4)$
$\mathbf{6_m}$	$\mathrm{Cov}(\mathbf{6},15)$	$\mathrm{Cov}(\mathbf{6},8)$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},11)$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},11)$

Table 4: Information on the indices k, l and m and the form $\mathrm{Cov}(\mathbf{6}, f)$ of the six-dimensional representations, contained in the products $\mathbf{3_i} \times \mathbf{6_j}$. For the products $\mathbf{3_i^-} \times \mathbf{6_j}$ use the covariants $\mathrm{Cov}(\mathbf{6}, f, -)$ instead of $\mathrm{Cov}(\mathbf{6}, f)$.

In the case of i = 4 the products $\mathbf{3_4} \times \mathbf{6_j}$ and $\mathbf{3_4^-} \times \mathbf{6_j}$ decompose either into four three-dimensional representations $\mathbf{3_k}$, $\mathbf{3_k^-}$, $\mathbf{3_l}$ and $\mathbf{3_l^-}$ and one six-dimensional one $\mathbf{6_m}$ or into three six-dimensional ones, $\mathbf{6_k}$, $\mathbf{6_l}$ and $\mathbf{6_m}$. These can be found in table 5.

j	3	4	6
k	3	1	2
l	7	5	6
$\mathbf{3_k}$	$\mathrm{Cov}(\mathbf{3},4)$	$\mathrm{Cov}(\mathbf{3},1)$	$\mathrm{Cov}(\mathbf{3},4)$
$\mathbf{3_k^-}$	$\mathrm{Cov}(\mathbf{3},4,-)$	$\mathrm{Cov}(\mathbf{3},1,-)$	$\mathrm{Cov}(\mathbf{3},4,-)$
$\mathbf{3_l}$	$\mathrm{Cov}(\mathbf{3},3)$	$\mathrm{Cov}(\mathbf{3},5)$	$\mathrm{Cov}(\mathbf{3},3)$
$\mathbf{3_l^-}$	$\mathrm{Cov}(\mathbf{3},3,-)$	$\mathrm{Cov}(\mathbf{3},5,-)$	$\mathrm{Cov}(\mathbf{3},3,-)$
m	3	4	6
$\mathbf{6_m}$	$\mathrm{Cov}(\mathbf{6},8)$	$\mathrm{Cov}(\mathbf{6},10)$	$\mathrm{Cov}(\mathbf{6},8)$

j	1	2	5	7
k	2	1	1	1
l	5	5	2	2
m	7	7	7	5
$\mathbf{6_k}$	$\mathrm{Cov}(\mathbf{6},1)$	$\mathrm{Cov}(\mathbf{6},4)$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},14)$
$\mathbf{6_l}$	$\mathrm{Cov}(\mathbf{6},5)$	$\mathrm{Cov}(\mathbf{6},7)$	$\mathrm{Cov}(\mathbf{6},11)$	$\mathrm{Cov}(\mathbf{6},8)$
$\mathbf{6_m}$	$\mathrm{Cov}(\mathbf{6},15)$	$\mathrm{Cov}(\mathbf{6},8)$	$\mathrm{Cov}(\mathbf{6},10)$	$\mathrm{Cov}(\mathbf{6},10)$

Table 5: Information on the indices k, l and m and the forms $\mathrm{Cov}(\mathbf{3}, f)$, $\mathrm{Cov}(\mathbf{3}, f, -)$ and $\mathrm{Cov}(\mathbf{6}, f)$ of the three- and six-dimensional representations, contained in the products $\mathbf{3_4} \times \mathbf{6_j}$. For the products $\mathbf{3_4^-} \times \mathbf{6_j}$ exchange the covariants $\mathrm{Cov}(\mathbf{3}, f)$ and $\mathrm{Cov}(\mathbf{3}, f, -)$ with each other and use the covariants $\mathrm{Cov}(\mathbf{6}, f, -)$ instead of $\mathrm{Cov}(\mathbf{6}, f)$.

The covariants $\mathrm{Cov}(\mathbf{3}, f)$ are given by

$$\mathrm{Cov}(\mathbf{3},1): \begin{pmatrix} y_3 z_2 + y_2 z_4 \\ y_1 z_3 + y_3 z_6 \\ y_2 z_1 + y_1 z_5 \end{pmatrix}, \quad \mathrm{Cov}(\mathbf{3},2): \begin{pmatrix} y_3 z_3 + y_2 z_5 \\ y_1 z_1 + y_3 z_4 \\ y_2 z_2 + y_1 z_6 \end{pmatrix}, \quad \mathrm{Cov}(\mathbf{3},3): \begin{pmatrix} y_2 z_1 + y_3 z_6 \\ y_3 z_2 + y_1 z_5 \\ y_1 z_3 + y_2 z_4 \end{pmatrix},$$
$$\mathrm{Cov}(\mathbf{3},4): \begin{pmatrix} y_3 z_1 + y_2 z_6 \\ y_1 z_2 + y_3 z_5 \\ y_2 z_3 + y_1 z_4 \end{pmatrix}, \quad \mathrm{Cov}(\mathbf{3},5): \begin{pmatrix} y_2 z_2 + y_3 z_4 \\ y_3 z_3 + y_1 z_6 \\ y_1 z_1 + y_2 z_5 \end{pmatrix}, \quad \mathrm{Cov}(\mathbf{3},6): \begin{pmatrix} y_2 z_3 + y_3 z_5 \\ y_3 z_1 + y_1 z_4 \\ y_1 z_2 + y_2 z_6 \end{pmatrix} \qquad (16)$$

and Cov$(\mathbf{3}, f, -)$ have the same form with a relative minus sign among the two terms in the different components.
The different forms of the covariants, forming six-dimensional representations, are as follows

$$
\begin{array}{llll}
\mathrm{Cov}(\mathbf{6},1) \; : \; \begin{pmatrix} y_2\, z_3 \\ y_3\, z_1 \\ y_1\, z_2 \\ y_2\, z_6 \\ y_1\, z_4 \\ y_3\, z_5 \end{pmatrix} , &
\mathrm{Cov}(\mathbf{6},2) \; : \; \begin{pmatrix} y_3\, z_4 \\ y_1\, z_6 \\ y_2\, z_5 \\ y_1\, z_1 \\ y_3\, z_3 \\ y_2\, z_2 \end{pmatrix} , &
\mathrm{Cov}(\mathbf{6},3) \; : \; \begin{pmatrix} y_1\, z_4 \\ y_2\, z_6 \\ y_3\, z_5 \\ y_3\, z_1 \\ y_2\, z_3 \\ y_1\, z_2 \end{pmatrix} , &
\mathrm{Cov}(\mathbf{6},4) \; : \; \begin{pmatrix} y_3\, z_2 \\ y_1\, z_3 \\ y_2\, z_1 \\ y_1\, z_5 \\ y_3\, z_6 \\ y_2\, z_4 \end{pmatrix} , \\
\mathrm{Cov}(\mathbf{6},5) \; : \; \begin{pmatrix} y_2\, z_1 \\ y_3\, z_2 \\ y_1\, z_3 \\ y_2\, z_4 \\ y_1\, z_5 \\ y_3\, z_6 \end{pmatrix} , &
\mathrm{Cov}(\mathbf{6},6) \; : \; \begin{pmatrix} y_3\, z_1 \\ y_1\, z_2 \\ y_2\, z_3 \\ y_1\, z_4 \\ y_3\, z_5 \\ y_2\, z_6 \end{pmatrix} , &
\mathrm{Cov}(\mathbf{6},7) \; : \; \begin{pmatrix} y_1\, z_2 \\ y_2\, z_3 \\ y_3\, z_1 \\ y_3\, z_5 \\ y_2\, z_6 \\ y_1\, z_4 \end{pmatrix} , &
\mathrm{Cov}(\mathbf{6},8) \; : \; \begin{pmatrix} y_1\, z_6 \\ y_2\, z_5 \\ y_3\, z_4 \\ y_3\, z_3 \\ y_2\, z_2 \\ y_1\, z_1 \end{pmatrix} , \\
\mathrm{Cov}(\mathbf{6},9) \; : \; \begin{pmatrix} y_1\, z_3 \\ y_2\, z_1 \\ y_3\, z_2 \\ y_3\, z_6 \\ y_2\, z_4 \\ y_1\, z_5 \end{pmatrix} , &
\mathrm{Cov}(\mathbf{6},10) \; : \; \begin{pmatrix} y_3\, z_5 \\ y_1\, z_4 \\ y_2\, z_6 \\ y_1\, z_2 \\ y_3\, z_1 \\ y_2\, z_3 \end{pmatrix} , &
\mathrm{Cov}(\mathbf{6},11) \; : \; \begin{pmatrix} y_3\, z_3 \\ y_1\, z_1 \\ y_2\, z_2 \\ y_1\, z_6 \\ y_3\, z_4 \\ y_2\, z_5 \end{pmatrix} , &
\mathrm{Cov}(\mathbf{6},12) \; : \; \begin{pmatrix} y_2\, z_2 \\ y_3\, z_3 \\ y_1\, z_1 \\ y_2\, z_5 \\ y_1\, z_6 \\ y_3\, z_4 \end{pmatrix} , \\
\mathrm{Cov}(\mathbf{6},13) \; : \; \begin{pmatrix} y_1\, z_1 \\ y_2\, z_2 \\ y_3\, z_3 \\ y_3\, z_4 \\ y_2\, z_5 \\ y_1\, z_6 \end{pmatrix} , &
\mathrm{Cov}(\mathbf{6},14) \; : \; \begin{pmatrix} y_1\, z_5 \\ y_2\, z_4 \\ y_3\, z_6 \\ y_3\, z_2 \\ y_2\, z_1 \\ y_1\, z_3 \end{pmatrix} , &
\mathrm{Cov}(\mathbf{6},15) \; : \; \begin{pmatrix} y_2\, z_5 \\ y_3\, z_4 \\ y_1\, z_6 \\ y_2\, z_2 \\ y_1\, z_1 \\ y_3\, z_3 \end{pmatrix} . &
\end{array}
\tag{17}
$$

The covariants Cov$(\mathbf{6}, f, -)$ are of the same form as Cov$(\mathbf{6}, f)$ with the last three components acquiring a minus sign.

Lastly, we also present the Clebsch Gordan coefficients for the products $\mathbf{6_i} \times \mathbf{6_j}$ with i $\leq$ j. We first consider those which contain the trivial representation $\mathbf{1}$. We find for $y_a \sim \mathbf{6_i}$ and $z_b \sim \mathbf{6_j}$ for these

$$
\begin{array}{rcl}
\mathbf{6_1} \times \mathbf{6_1} &=& \mathbf{1}\,((\mathbf{1})_1) + \mathbf{1}^-\,((\mathbf{1}^-)_1) + 2\,\mathbf{2}\,((\mathbf{2})^{\mathrm{I}}_1,\,(\mathbf{2})^{\mathrm{I}}_2) + \mathbf{3_3}\,(\mathrm{Cov}(\mathbf{3},7)) + \mathbf{3_3^-}\,(\mathrm{Cov}(\mathbf{3},7,-)) \\
&& +\mathbf{3_5}\,(\mathrm{Cov}(\mathbf{3},8)) + \mathbf{3_5^-}\,(\mathrm{Cov}(\mathbf{3},8,-)) + 2\,\mathbf{6_1}\,((\mathbf{6})^{\mathrm{I}}_1,\,(\mathbf{6})^{\mathrm{I}}_2) + \mathbf{6_6}\,(\mathrm{Cov}(\mathbf{6},16)) \\
\mathbf{6_2} \times \mathbf{6_5} &=& \mathbf{1}\,((\mathbf{1})_2) + \mathbf{1}^-\,((\mathbf{1}^-)_2) + 2\,\mathbf{2}\,((\mathbf{2})^{\mathrm{II}}_1,\,(\mathbf{2})^{\mathrm{II}}_2) + \mathbf{6_1}\,(\mathrm{Cov}(\mathbf{6},17)) + \mathbf{6_3}\,(\mathrm{Cov}(\mathbf{6},18)) \\
&& +\mathbf{6_4}\,(\mathrm{Cov}(\mathbf{6},19)) + \mathbf{6_6}\,(\mathrm{Cov}(\mathbf{6},20)) + \mathbf{6_7}\,(\mathrm{Cov}(\mathbf{6},21)) \\
\mathbf{6_3} \times \mathbf{6_4} &=& \mathbf{1}\,((\mathbf{1})_2) + \mathbf{1}^-\,((\mathbf{1}^-)_2) + 2\,\mathbf{2}\,((\mathbf{2})^{\mathrm{II}}_1,\,(\mathbf{2})^{\mathrm{II}}_2) + \mathbf{3_4}\,(\mathrm{Cov}(\mathbf{3},9)) + \mathbf{3_4^-}\,(\mathrm{Cov}(\mathbf{3},9,-)) \\
&& +\mathbf{6_1}\,(\mathrm{Cov}(\mathbf{6},17)) + \mathbf{6_2}\,(\mathrm{Cov}(\mathbf{6},22)) + \mathbf{6_5}\,(\mathrm{Cov}(\mathbf{6},23)) + \mathbf{6_7}\,(\mathrm{Cov}(\mathbf{6},24)) \\
\mathbf{6_6} \times \mathbf{6_6} &=& \mathbf{1}\,((\mathbf{1})_1) + \mathbf{1}^-\,((\mathbf{1}^-)_1) + 2\,\mathbf{2}\,((\mathbf{2})^{\mathrm{I}}_1,\,(\mathbf{2})^{\mathrm{I}}_2) + \mathbf{3_2}\,(\mathrm{Cov}(\mathbf{3},8)) + \mathbf{3_2^-}\,(\mathrm{Cov}(\mathbf{3},8,-)) \\
&& +\mathbf{3_4}\,(\mathrm{Cov}(\mathbf{3},10)) + \mathbf{3_4^-}\,(\mathrm{Cov}(\mathbf{3},10,-)) + \mathbf{3_6}\,(\mathrm{Cov}(\mathbf{3},7)) + \mathbf{3_6^-}\,(\mathrm{Cov}(\mathbf{3},7,-)) + 2\,\mathbf{6_6}\,((\mathbf{6})^{\mathrm{I}}_1,\,(\mathbf{6})^{\mathrm{I}}_2) \\
\mathbf{6_7} \times \mathbf{6_7} &=& \mathbf{1}\,((\mathbf{1})_2) + \mathbf{1}^-\,((\mathbf{1}^-)_2) + 2\,\mathbf{2}\,((\mathbf{2})^{\mathrm{II}}_1,\,(\mathbf{2})^{\mathrm{II}}_2) + \mathbf{3_1}\,(\mathrm{Cov}(\mathbf{3},11)) + \mathbf{3_1^-}\,(\mathrm{Cov}(\mathbf{3},11,-)) \\
&& +\mathbf{3_7}\,(\mathrm{Cov}(\mathbf{3},7)) + \mathbf{3_7^-}\,(\mathrm{Cov}(\mathbf{3},7,-)) + \mathbf{6_6}\,(\mathrm{Cov}(\mathbf{6},17)) + 2\,\mathbf{6_7}\,((\mathbf{6})^{\mathrm{II}}_1,\,(\mathbf{6})^{\mathrm{II}}_2)
\end{array}
\tag{18}
$$

with the form of the covariants denoted in brackets being

$$
\begin{array}{ll}
(\mathbf{1})_1: & y_1 z_6 + y_2 z_5 + y_3 z_4 + y_4 z_3 + y_5 z_2 + y_6 z_1 \sim \mathbf{1}\,, \\
(\mathbf{1})_2: & y_1 z_4 + y_2 z_6 + y_3 z_5 + y_4 z_1 + y_5 z_3 + y_6 z_2 \sim \mathbf{1}\,, \\
(\mathbf{1}^-)_1: & y_1 z_6 + y_2 z_5 + y_3 z_4 - y_4 z_3 - y_5 z_2 - y_6 z_1 \sim \mathbf{1}^-\,, \\
(\mathbf{1}^-)_2: & y_1 z_4 + y_2 z_6 + y_3 z_5 - y_4 z_1 - y_5 z_3 - y_6 z_2 \sim \mathbf{1}^-\,, \\
(\mathbf{2})^{\rm I}_1: & \left(\begin{array}{c} \omega\,(-y_1 z_6 + y_6 z_1) - y_2 z_5 + y_5 z_2 + \omega^2\,(-y_3 z_4 + y_4 z_3) \\ \omega^2\,(y_1 z_6 - y_6 z_1) + y_2 z_5 - y_5 z_2 + \omega\,(y_3 z_4 - y_4 z_3) \end{array} \right) \sim \mathbf{2}\,, \\
(\mathbf{2})^{\rm I}_2: & \left(\begin{array}{c} \omega\,(y_1 z_6 + y_6 z_1) + y_2 z_5 + y_5 z_2 + \omega^2\,(y_3 z_4 + y_4 z_3) \\ \omega^2\,(y_1 z_6 + y_6 z_1) + y_2 z_5 + y_5 z_2 + \omega\,(y_3 z_4 + y_4 z_3) \end{array} \right) \sim \mathbf{2}\,, \\
(\mathbf{2})^{\rm II}_1: & \left(\begin{array}{c} \omega\,(-y_3 z_5 + y_5 z_3) - y_1 z_4 + y_4 z_1 + \omega^2\,(-y_2 z_6 + y_6 z_2) \\ \omega^2\,(y_3 z_5 - y_5 z_3) + y_1 z_4 - y_4 z_1 + \omega\,(y_2 z_6 - y_6 z_2) \end{array} \right) \sim \mathbf{2}\,, \\
(\mathbf{2})^{\rm II}_2: & \left(\begin{array}{c} \omega\,(y_3 z_5 + y_5 z_3) + y_1 z_4 + y_4 z_1 + \omega^2\,(y_2 z_6 + y_6 z_2) \\ \omega^2\,(y_3 z_5 + y_5 z_3) + y_1 z_4 + y_4 z_1 + \omega\,(y_2 z_6 + y_6 z_2) \end{array} \right) \sim \mathbf{2}
\end{array}
\qquad (19)
$$

and see Eqs. (25)-(27). The remaining products $\mathbf{6_i}\times\mathbf{6_j}$ with $\rm i \leq j$ decompose into a certain number of irreducible three-dimensional and six-dimensional representations. They are listed in the following together with the form of the covariants given in brackets. We always assume $y_a \sim \mathbf{6_i}$ and $z_b \sim \mathbf{6_j}$.

$$
\begin{array}{lcl}
\mathbf{6_1} \times \mathbf{6_2} &=& \mathbf{3_2}\,({\rm Cov}(\mathbf{3},9)) + \mathbf{3_2^-}\,({\rm Cov}(\mathbf{3},9,-)) + \mathbf{3_4}\,({\rm Cov}(\mathbf{3},8)) + \mathbf{3_4^-}\,({\rm Cov}(\mathbf{3},8,-)) \\
&& +\mathbf{3_7}\,({\rm Cov}(\mathbf{3},12)) + \mathbf{3_7^-}\,({\rm Cov}(\mathbf{3},12,-)) + \mathbf{6_2}\,({\rm Cov}(\mathbf{6},25)) + \mathbf{6_4}\,({\rm Cov}(\mathbf{6},22)) + \mathbf{6_7}\,({\rm Cov}(\mathbf{6},16)) \\
\mathbf{6_1} \times \mathbf{6_3} &=& \mathbf{3_3}\,({\rm Cov}(\mathbf{3},8)) + \mathbf{3_3^-}\,({\rm Cov}(\mathbf{3},8,-)) + \mathbf{3_6}\,({\rm Cov}(\mathbf{3},12)) + \mathbf{3_6^-}\,({\rm Cov}(\mathbf{3},12,-)) \\
&& +\mathbf{6_3}\,({\rm Cov}(\mathbf{6},25)) + \mathbf{6_5}\,({\rm Cov}(\mathbf{6},21)) + \mathbf{6_6}\,({\rm Cov}(\mathbf{6},26)) + \mathbf{6_7}\,({\rm Cov}(\mathbf{6},22)) \\
\mathbf{6_1} \times \mathbf{6_4} &=& \mathbf{3_2}\,({\rm Cov}(\mathbf{3},8)) + \mathbf{3_2^-}\,({\rm Cov}(\mathbf{3},8,-)) + \mathbf{3_5}\,({\rm Cov}(\mathbf{3},12)) + \mathbf{3_5^-}\,({\rm Cov}(\mathbf{3},12,-)) \\
&& +\mathbf{6_2}\,({\rm Cov}(\mathbf{6},26)) + \mathbf{6_4}\,({\rm Cov}(\mathbf{6},25)) + \mathbf{6_6}\,({\rm Cov}(\mathbf{6},27)) + \mathbf{6_7}\,({\rm Cov}(\mathbf{6},19)) \\
\mathbf{6_1} \times \mathbf{6_5} &=& \mathbf{3_1}\,({\rm Cov}(\mathbf{3},8)) + \mathbf{3_1^-}\,({\rm Cov}(\mathbf{3},8,-)) + \mathbf{3_4}\,({\rm Cov}(\mathbf{3},12)) + \mathbf{3_4^-}\,({\rm Cov}(\mathbf{3},12,-)) \\
&& +\mathbf{3_6}\,({\rm Cov}(\mathbf{3},10)) + \mathbf{3_6^-}\,({\rm Cov}(\mathbf{3},10,-)) + \mathbf{6_3}\,({\rm Cov}(\mathbf{6},19)) + \mathbf{6_5}\,({\rm Cov}(\mathbf{6},25)) + \mathbf{6_7}\,({\rm Cov}(\mathbf{6},27)) \\
\mathbf{6_1} \times \mathbf{6_6} &=& \mathbf{3_3}\,({\rm Cov}(\mathbf{3},9)) + \mathbf{3_3^-}\,({\rm Cov}(\mathbf{3},9,-)) + \mathbf{3_5}\,({\rm Cov}(\mathbf{3},13)) + \mathbf{3_5^-}\,({\rm Cov}(\mathbf{3},13,-)) \\
&& +\mathbf{6_1}\,({\rm Cov}(\mathbf{6},28)) + \mathbf{6_3}\,({\rm Cov}(\mathbf{6},22)) + \mathbf{6_4}\,({\rm Cov}(\mathbf{6},29)) + \mathbf{6_7}\,({\rm Cov}(\mathbf{6},26)) \\
\mathbf{6_1} \times \mathbf{6_7} &=& \mathbf{3_4}\,({\rm Cov}(\mathbf{3},13)) + \mathbf{3_4^-}\,({\rm Cov}(\mathbf{3},13,-)) + \mathbf{6_2}\,({\rm Cov}(\mathbf{6},28)) + \mathbf{6_3}\,({\rm Cov}(\mathbf{6},26)) \\
&& +\mathbf{6_4}\,({\rm Cov}(\mathbf{6},21)) + \mathbf{6_5}\,({\rm Cov}(\mathbf{6},29)) + \mathbf{6_6}\,({\rm Cov}(\mathbf{6},22))
\end{array}
\qquad (20)
$$

$$
\begin{array}{lcl}
\mathbf{6_2} \times \mathbf{6_2} &=& \mathbf{3_1}\,({\rm Cov}(\mathbf{3},11)) + \mathbf{3_1^-}\,({\rm Cov}(\mathbf{3},11,-)) + \mathbf{3_3}\,({\rm Cov}(\mathbf{3},8)) + \mathbf{3_3^-}\,({\rm Cov}(\mathbf{3},8,-)) \\
&& +\mathbf{3_4}\,({\rm Cov}(\mathbf{3},7)) + \mathbf{3_4^-}\,({\rm Cov}(\mathbf{3},7,-)) + 2\,\mathbf{6_5}\,((\mathbf{6})^{\rm II}_1,\,(\mathbf{6})^{\rm II}_2) + \mathbf{6_6}\,({\rm Cov}(\mathbf{6},26)) \\
\mathbf{6_2} \times \mathbf{6_3} &=& \mathbf{3_2}\,({\rm Cov}(\mathbf{3},8)) + \mathbf{3_2^-}\,({\rm Cov}(\mathbf{3},8,-)) + \mathbf{3_3}\,({\rm Cov}(\mathbf{3},9)) + \mathbf{3_3^-}\,({\rm Cov}(\mathbf{3},9,-)) \\
&& +\mathbf{6_1}\,({\rm Cov}(\mathbf{6},18)) + \mathbf{6_2}\,({\rm Cov}(\mathbf{6},26)) + \mathbf{6_3}\,({\rm Cov}(\mathbf{6},22)) + \mathbf{6_6}\,({\rm Cov}(\mathbf{6},25)) \\
\mathbf{6_2} \times \mathbf{6_4} &=& \mathbf{3_1}\,({\rm Cov}(\mathbf{3},8)) + \mathbf{3_1^-}\,({\rm Cov}(\mathbf{3},8,-)) + \mathbf{3_6}\,({\rm Cov}(\mathbf{3},10)) + \mathbf{3_6^-}\,({\rm Cov}(\mathbf{3},10,-)) \\
&& +\mathbf{6_2}\,({\rm Cov}(\mathbf{6},18)) + \mathbf{6_4}\,({\rm Cov}(\mathbf{6},21)) + \mathbf{6_6}\,({\rm Cov}(\mathbf{6},22)) + \mathbf{6_7}\,({\rm Cov}(\mathbf{6},25)) \\
\mathbf{6_2} \times \mathbf{6_6} &=& \mathbf{3_5}\,({\rm Cov}(\mathbf{3},7)) + \mathbf{3_5^-}\,({\rm Cov}(\mathbf{3},7,-)) + \mathbf{3_7}\,({\rm Cov}(\mathbf{3},12)) + \mathbf{3_7^-}\,({\rm Cov}(\mathbf{3},12,-)) \\
&& +\mathbf{6_2}\,({\rm Cov}(\mathbf{6},25)) + \mathbf{6_3}\,({\rm Cov}(\mathbf{6},26)) + \mathbf{6_4}\,({\rm Cov}(\mathbf{6},30)) + \mathbf{6_5}\,({\rm Cov}(\mathbf{6},29)) \\
\mathbf{6_2} \times \mathbf{6_7} &=& \mathbf{3_4}\,({\rm Cov}(\mathbf{3},9)) + \mathbf{3_4^-}\,({\rm Cov}(\mathbf{3},9,-)) + \mathbf{3_5}\,({\rm Cov}(\mathbf{3},10)) + \mathbf{3_5^-}\,({\rm Cov}(\mathbf{3},10,-)) \\
&& +\mathbf{3_6}\,({\rm Cov}(\mathbf{3},12)) + \mathbf{3_6^-}\,({\rm Cov}(\mathbf{3},12,-)) + \mathbf{6_1}\,({\rm Cov}(\mathbf{6},31)) + \mathbf{6_2}\,({\rm Cov}(\mathbf{6},22)) + \mathbf{6_3}\,({\rm Cov}(\mathbf{6},25))
\end{array}
\qquad (21)
$$

$$\begin{aligned}
\mathbf{6_3}\times\mathbf{6_3} &= \mathbf{3_1}\,(\mathrm{Cov}(\mathbf{3},8)) + \mathbf{3_1^-}\,(\mathrm{Cov}(\mathbf{3},8,-)) + \mathbf{3_2}\,(\mathrm{Cov}(\mathbf{3},11)) + \mathbf{3_2^-}\,(\mathrm{Cov}(\mathbf{3},11,-))\\
&\quad + \mathbf{3_5}\,(\mathrm{Cov}(\mathbf{3},7)) + \mathbf{3_5^-}\,(\mathrm{Cov}(\mathbf{3},7,-)) + \mathbf{3_6}\,(\mathrm{Cov}(\mathbf{3},10)) + \mathbf{3_6^-}\,(\mathrm{Cov}(\mathbf{3},10,-))\\
&\quad + 2\,\mathbf{6_4}\,((\mathbf{6})_1^{\mathrm{II}},\,(\mathbf{6})_2^{\mathrm{II}})\\
\mathbf{6_3}\times\mathbf{6_5} &= \mathbf{3_2}\,(\mathrm{Cov}(\mathbf{3},14)) + \mathbf{3_2^-}\,(\mathrm{Cov}(\mathbf{3},14,-)) + \mathbf{3_7}\,(\mathrm{Cov}(\mathbf{3},8)) + \mathbf{3_7^-}\,(\mathrm{Cov}(\mathbf{3},8,-))\\
&\quad + \mathbf{6_3}\,(\mathrm{Cov}(\mathbf{6},21)) + \mathbf{6_5}\,(\mathrm{Cov}(\mathbf{6},19)) + \mathbf{6_6}\,(\mathrm{Cov}(\mathbf{6},18)) + \mathbf{6_7}\,(\mathrm{Cov}(\mathbf{6},20))\\
\mathbf{6_3}\times\mathbf{6_6} &= \mathbf{3_1}\,(\mathrm{Cov}(\mathbf{3},11)) + \mathbf{3_1^-}\,(\mathrm{Cov}(\mathbf{3},11,-)) + \mathbf{3_5}\,(\mathrm{Cov}(\mathbf{3},10)) + \mathbf{3_5^-}\,(\mathrm{Cov}(\mathbf{3},10,-))\\
&\quad + \mathbf{6_1}\,(\mathrm{Cov}(\mathbf{6},31)) + \mathbf{6_2}\,(\mathrm{Cov}(\mathbf{6},32)) + \mathbf{6_5}\,(\mathrm{Cov}(\mathbf{6},24)) + \mathbf{6_7}\,(\mathrm{Cov}(\mathbf{6},30))\\
\mathbf{6_3}\times\mathbf{6_7} &= \mathbf{3_6}\,(\mathrm{Cov}(\mathbf{3},7)) + \mathbf{3_6^-}\,(\mathrm{Cov}(\mathbf{3},7,-)) + \mathbf{3_7}\,(\mathrm{Cov}(\mathbf{3},15)) + \mathbf{3_7^-}\,(\mathrm{Cov}(\mathbf{3},15,-))\\
&\quad + \mathbf{6_1}\,(\mathrm{Cov}(\mathbf{6},18)) + \mathbf{6_2}\,(\mathrm{Cov}(\mathbf{6},17)) + \mathbf{6_3}\,(\mathrm{Cov}(\mathbf{6},30)) + \mathbf{6_6}\,(\mathrm{Cov}(\mathbf{6},25))
\end{aligned}\tag{22}$$

$$\begin{aligned}
\mathbf{6_4}\times\mathbf{6_4} &= \mathbf{3_2}\,(\mathrm{Cov}(\mathbf{3},14)) + \mathbf{3_2^-}\,(\mathrm{Cov}(\mathbf{3},14,-)) + \mathbf{3_3}\,(\mathrm{Cov}(\mathbf{3},11)) + \mathbf{3_3^-}\,(\mathrm{Cov}(\mathbf{3},11,-))\\
&\quad + \mathbf{3_6}\,(\mathrm{Cov}(\mathbf{3},7)) + \mathbf{3_6^-}\,(\mathrm{Cov}(\mathbf{3},7,-)) + \mathbf{3_7}\,(\mathrm{Cov}(\mathbf{3},8)) + \mathbf{3_7^-}\,(\mathrm{Cov}(\mathbf{3},8,-))\\
&\quad + 2\,\mathbf{6_3}\,((\mathbf{6})_1^{\mathrm{II}},\,(\mathbf{6})_2^{\mathrm{II}})\\
\mathbf{6_4}\times\mathbf{6_5} &= \mathbf{3_5}\,(\mathrm{Cov}(\mathbf{3},9)) + \mathbf{3_5^-}\,(\mathrm{Cov}(\mathbf{3},9,-)) + \mathbf{3_6}\,(\mathrm{Cov}(\mathbf{3},8)) + \mathbf{3_6^-}\,(\mathrm{Cov}(\mathbf{3},8,-))\\
&\quad + \mathbf{6_1}\,(\mathrm{Cov}(\mathbf{6},22)) + \mathbf{6_4}\,(\mathrm{Cov}(\mathbf{6},23)) + \mathbf{6_5}\,(\mathrm{Cov}(\mathbf{6},33)) + \mathbf{6_6}\,(\mathrm{Cov}(\mathbf{6},24))\\
\mathbf{6_4}\times\mathbf{6_6} &= \mathbf{3_3}\,(\mathrm{Cov}(\mathbf{3},16)) + \mathbf{3_3^-}\,(\mathrm{Cov}(\mathbf{3},16,-)) + \mathbf{3_7}\,(\mathrm{Cov}(\mathbf{3},15)) + \mathbf{3_7^-}\,(\mathrm{Cov}(\mathbf{3},15,-))\\
&\quad + \mathbf{6_1}\,(\mathrm{Cov}(\mathbf{6},34)) + \mathbf{6_2}\,(\mathrm{Cov}(\mathbf{6},17)) + \mathbf{6_5}\,(\mathrm{Cov}(\mathbf{6},35)) + \mathbf{6_7}\,(\mathrm{Cov}(\mathbf{6},36))\\
\mathbf{6_4}\times\mathbf{6_7} &= \mathbf{3_1}\,(\mathrm{Cov}(\mathbf{3},17)) + \mathbf{3_1^-}\,(\mathrm{Cov}(\mathbf{3},17,-)) + \mathbf{3_2}\,(\mathrm{Cov}(\mathbf{3},11)) + \mathbf{3_2^-}\,(\mathrm{Cov}(\mathbf{3},11,-))\\
&\quad + \mathbf{6_1}\,(\mathrm{Cov}(\mathbf{6},32)) + \mathbf{6_4}\,(\mathrm{Cov}(\mathbf{6},24)) + \mathbf{6_5}\,(\mathrm{Cov}(\mathbf{6},37)) + \mathbf{6_6}\,(\mathrm{Cov}(\mathbf{6},30))
\end{aligned}\tag{23}$$

$$\begin{aligned}
\mathbf{6_5}\times\mathbf{6_5} &= \mathbf{3_4}\,(\mathrm{Cov}(\mathbf{3},11)) + \mathbf{3_4^-}\,(\mathrm{Cov}(\mathbf{3},11,-)) + \mathbf{3_5}\,(\mathrm{Cov}(\mathbf{3},8)) + \mathbf{3_5^-}\,(\mathrm{Cov}(\mathbf{3},8,-))\\
&\quad + \mathbf{3_7}\,(\mathrm{Cov}(\mathbf{3},7)) + \mathbf{3_7^-}\,(\mathrm{Cov}(\mathbf{3},7,-)) + 2\,\mathbf{6_2}\,((\mathbf{6})_1^{\mathrm{II}},\,(\mathbf{6})_2^{\mathrm{II}}) + \mathbf{6_6}\,(\mathrm{Cov}(\mathbf{6},16))\\
\mathbf{6_5}\times\mathbf{6_6} &= \mathbf{3_1}\,(\mathrm{Cov}(\mathbf{3},17)) + \mathbf{3_1^-}\,(\mathrm{Cov}(\mathbf{3},17,-)) + \mathbf{3_3}\,(\mathrm{Cov}(\mathbf{3},18)) + \mathbf{3_3^-}\,(\mathrm{Cov}(\mathbf{3},18,-))\\
&\quad + \mathbf{6_2}\,(\mathrm{Cov}(\mathbf{6},34)) + \mathbf{6_3}\,(\mathrm{Cov}(\mathbf{6},36)) + \mathbf{6_4}\,(\mathrm{Cov}(\mathbf{6},38)) + \mathbf{6_5}\,(\mathrm{Cov}(\mathbf{6},37))\\
\mathbf{6_5}\times\mathbf{6_7} &= \mathbf{3_2}\,(\mathrm{Cov}(\mathbf{3},18)) + \mathbf{3_2^-}\,(\mathrm{Cov}(\mathbf{3},18,-)) + \mathbf{3_3}\,(\mathrm{Cov}(\mathbf{3},14)) + \mathbf{3_3^-}\,(\mathrm{Cov}(\mathbf{3},14,-))\\
&\quad + \mathbf{3_4}\,(\mathrm{Cov}(\mathbf{3},16)) + \mathbf{3_4^-}\,(\mathrm{Cov}(\mathbf{3},16,-)) + \mathbf{6_1}\,(\mathrm{Cov}(\mathbf{6},29)) + \mathbf{6_4}\,(\mathrm{Cov}(\mathbf{6},36)) + \mathbf{6_5}\,(\mathrm{Cov}(\mathbf{6},39))\\
\mathbf{6_6}\times\mathbf{6_7} &= \mathbf{3_1}\,(\mathrm{Cov}(\mathbf{3},8)) + \mathbf{3_1^-}\,(\mathrm{Cov}(\mathbf{3},8,-)) + \mathbf{3_7}\,(\mathrm{Cov}(\mathbf{3},12)) + \mathbf{3_7^-}\,(\mathrm{Cov}(\mathbf{3},12,-))\\
&\quad + \mathbf{6_1}\,(\mathrm{Cov}(\mathbf{6},22)) + \mathbf{6_3}\,(\mathrm{Cov}(\mathbf{6},17)) + \mathbf{6_4}\,(\mathrm{Cov}(\mathbf{6},30)) + \mathbf{6_7}\,(\mathrm{Cov}(\mathbf{6},25))
\end{aligned}\tag{24}$$

with the covariants $\mathrm{Cov}(\mathbf{3},f)$ and $\mathrm{Cov}(\mathbf{6},f)$ being found in Eqs. (25)-(27).

The covariants $\mathrm{Cov}(\mathbf{3},f)$ read

$$\begin{aligned}
&\mathrm{Cov}(\mathbf{3},7):\begin{pmatrix} y_2 z_4 + y_4 z_2\\ y_3 z_6 + y_6 z_3\\ y_1 z_5 + y_5 z_1\end{pmatrix}, \quad \mathrm{Cov}(\mathbf{3},8):\begin{pmatrix} y_3 z_5 + y_5 z_3\\ y_1 z_4 + y_4 z_1\\ y_2 z_6 + y_6 z_2\end{pmatrix}, \quad \mathrm{Cov}(\mathbf{3},9):\begin{pmatrix} y_1 z_2 + y_6 z_4\\ y_2 z_3 + y_5 z_6\\ y_3 z_1 + y_4 z_5\end{pmatrix},\\
&\mathrm{Cov}(\mathbf{3},10):\begin{pmatrix} y_1 z_1 + y_6 z_6\\ y_2 z_2 + y_5 z_5\\ y_3 z_3 + y_4 z_4\end{pmatrix}, \quad \mathrm{Cov}(\mathbf{3},11):\begin{pmatrix} y_1 z_6 + y_6 z_1\\ y_2 z_5 + y_5 z_2\\ y_3 z_4 + y_4 z_3\end{pmatrix}, \quad \mathrm{Cov}(\mathbf{3},12):\begin{pmatrix} y_2 z_5 + y_4 z_3\\ y_3 z_4 + y_6 z_1\\ y_1 z_6 + y_5 z_2\end{pmatrix},\\
&\mathrm{Cov}(\mathbf{3},13):\begin{pmatrix} y_1 z_3 + y_6 z_5\\ y_2 z_1 + y_5 z_4\\ y_3 z_2 + y_4 z_6\end{pmatrix}, \quad \mathrm{Cov}(\mathbf{3},14):\begin{pmatrix} y_2 z_2 + y_4 z_4\\ y_3 z_3 + y_6 z_6\\ y_1 z_1 + y_5 z_5\end{pmatrix}, \quad \mathrm{Cov}(\mathbf{3},15):\begin{pmatrix} y_2 z_6 + y_4 z_1\\ y_3 z_5 + y_6 z_2\\ y_1 z_4 + y_5 z_3\end{pmatrix},\\
&\mathrm{Cov}(\mathbf{3},16):\begin{pmatrix} y_2 z_1 + y_4 z_6\\ y_3 z_2 + y_6 z_5\\ y_1 z_3 + y_5 z_4\end{pmatrix}, \quad \mathrm{Cov}(\mathbf{3},17):\begin{pmatrix} y_1 z_4 + y_6 z_2\\ y_2 z_6 + y_5 z_3\\ y_3 z_5 + y_4 z_1\end{pmatrix}, \quad \mathrm{Cov}(\mathbf{3},18):\begin{pmatrix} y_1 z_5 + y_6 z_3\\ y_2 z_4 + y_5 z_1\\ y_3 z_6 + y_4 z_2\end{pmatrix}
\end{aligned}\tag{25}$$

and $\mathrm{Cov}(\mathbf{3},f,-)$ have the same form with the relative sign between the two terms in the components being negative.

The covariants $(\mathbf{6})^{\rm I}_1$, $(\mathbf{6})^{\rm I}_2$, $(\mathbf{6})^{\rm II}_1$ and $(\mathbf{6})^{\rm II}_2$ are given by

$$(\mathbf{6})^{\rm I}_1 \;:\; \begin{pmatrix} y_4\,z_5+y_5\,z_4 \\ y_6\,z_4+y_4\,z_6 \\ y_5\,z_6+y_6\,z_5 \\ y_1\,z_2+y_2\,z_1 \\ y_3\,z_1+y_1\,z_3 \\ y_2\,z_3+y_3\,z_2 \end{pmatrix} \;,\quad (\mathbf{6})^{\rm I}_2 \;:\; \begin{pmatrix} y_4\,z_5-y_5\,z_4 \\ y_6\,z_4-y_4\,z_6 \\ y_5\,z_6-y_6\,z_5 \\ y_1\,z_2-y_2\,z_1 \\ y_3\,z_1-y_1\,z_3 \\ y_2\,z_3-y_3\,z_2 \end{pmatrix} \;,$$
$$(\mathbf{6})^{\rm II}_1 \;:\; \begin{pmatrix} y_5\,z_6+y_6\,z_5 \\ y_4\,z_5+y_5\,z_4 \\ y_6\,z_4+y_4\,z_6 \\ y_2\,z_3+y_3\,z_2 \\ y_1\,z_2+y_2\,z_1 \\ y_3\,z_1+y_1\,z_3 \end{pmatrix} \;,\quad (\mathbf{6})^{\rm II}_2 \;:\; \begin{pmatrix} y_5\,z_6-y_6\,z_5 \\ y_4\,z_5-y_5\,z_4 \\ y_6\,z_4-y_4\,z_6 \\ y_2\,z_3-y_3\,z_2 \\ y_1\,z_2-y_2\,z_1 \\ y_3\,z_1-y_1\,z_3 \end{pmatrix} \tag{26}$$

and the covariants $\mathrm{Cov}(\mathbf{6},f)$ read

$$\mathrm{Cov}(\mathbf{6},16) : \begin{pmatrix} y_1\,z_1 \\ y_2\,z_2 \\ y_3\,z_3 \\ y_4\,z_4 \\ y_5\,z_5 \\ y_6\,z_6 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},17) : \begin{pmatrix} y_5\,z_5 \\ y_4\,z_4 \\ y_6\,z_6 \\ y_2\,z_2 \\ y_1\,z_1 \\ y_3\,z_3 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},18) : \begin{pmatrix} y_3\,z_4 \\ y_1\,z_6 \\ y_2\,z_5 \\ y_6\,z_1 \\ y_4\,z_3 \\ y_5\,z_2 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},19) : \begin{pmatrix} y_4\,z_2 \\ y_6\,z_3 \\ y_5\,z_1 \\ y_1\,z_5 \\ y_3\,z_6 \\ y_2\,z_4 \end{pmatrix} ,$$
$$\mathrm{Cov}(\mathbf{6},20) : \begin{pmatrix} y_4\,z_6 \\ y_6\,z_5 \\ y_5\,z_4 \\ y_1\,z_3 \\ y_3\,z_2 \\ y_2\,z_1 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},21) : \begin{pmatrix} y_2\,z_3 \\ y_3\,z_1 \\ y_1\,z_2 \\ y_5\,z_6 \\ y_6\,z_4 \\ y_4\,z_5 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},22) : \begin{pmatrix} y_6\,z_3 \\ y_5\,z_1 \\ y_4\,z_2 \\ y_3\,z_6 \\ y_2\,z_4 \\ y_1\,z_5 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},23) : \begin{pmatrix} y_2\,z_5 \\ y_3\,z_4 \\ y_1\,z_6 \\ y_5\,z_2 \\ y_6\,z_1 \\ y_4\,z_3 \end{pmatrix} ,$$
$$\mathrm{Cov}(\mathbf{6},24) : \begin{pmatrix} y_6\,z_5 \\ y_5\,z_4 \\ y_4\,z_6 \\ y_3\,z_2 \\ y_2\,z_1 \\ y_1\,z_3 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},25) : \begin{pmatrix} y_5\,z_4 \\ y_4\,z_6 \\ y_6\,z_5 \\ y_2\,z_1 \\ y_1\,z_3 \\ y_3\,z_2 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},26) : \begin{pmatrix} y_2\,z_2 \\ y_3\,z_3 \\ y_1\,z_1 \\ y_5\,z_5 \\ y_6\,z_6 \\ y_4\,z_4 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},27) : \begin{pmatrix} y_3\,z_1 \\ y_1\,z_2 \\ y_2\,z_3 \\ y_6\,z_4 \\ y_4\,z_5 \\ y_5\,z_6 \end{pmatrix} ,$$
$$\mathrm{Cov}(\mathbf{6},28) : \begin{pmatrix} y_6\,z_1 \\ y_5\,z_2 \\ y_4\,z_3 \\ y_3\,z_4 \\ y_2\,z_5 \\ y_1\,z_6 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},29) : \begin{pmatrix} y_4\,z_1 \\ y_6\,z_2 \\ y_5\,z_3 \\ y_1\,z_4 \\ y_3\,z_5 \\ y_2\,z_6 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},30) : \begin{pmatrix} y_5\,z_6 \\ y_4\,z_5 \\ y_6\,z_4 \\ y_2\,z_3 \\ y_1\,z_2 \\ y_3\,z_1 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},31) : \begin{pmatrix} y_6\,z_2 \\ y_5\,z_3 \\ y_4\,z_1 \\ y_3\,z_5 \\ y_2\,z_6 \\ y_1\,z_4 \end{pmatrix} ,$$
$$\mathrm{Cov}(\mathbf{6},32) : \begin{pmatrix} y_3\,z_6 \\ y_1\,z_5 \\ y_2\,z_4 \\ y_6\,z_3 \\ y_4\,z_2 \\ y_5\,z_1 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},33) : \begin{pmatrix} y_3\,z_3 \\ y_1\,z_1 \\ y_2\,z_2 \\ y_6\,z_6 \\ y_4\,z_4 \\ y_5\,z_5 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},34) : \begin{pmatrix} y_4\,z_3 \\ y_6\,z_1 \\ y_5\,z_2 \\ y_1\,z_6 \\ y_3\,z_4 \\ y_2\,z_5 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},35) : \begin{pmatrix} y_2\,z_4 \\ y_3\,z_6 \\ y_1\,z_5 \\ y_5\,z_1 \\ y_6\,z_3 \\ y_4\,z_2 \end{pmatrix} ,$$
$$\mathrm{Cov}(\mathbf{6},36) : \begin{pmatrix} y_6\,z_4 \\ y_5\,z_6 \\ y_4\,z_5 \\ y_3\,z_1 \\ y_2\,z_3 \\ y_1\,z_2 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},37) : \begin{pmatrix} y_6\,z_6 \\ y_5\,z_5 \\ y_4\,z_4 \\ y_3\,z_3 \\ y_2\,z_2 \\ y_1\,z_1 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},38) : \begin{pmatrix} y_3\,z_2 \\ y_1\,z_3 \\ y_2\,z_1 \\ y_6\,z_5 \\ y_4\,z_6 \\ y_5\,z_4 \end{pmatrix} , \quad \mathrm{Cov}(\mathbf{6},39) : \begin{pmatrix} y_5\,z_2 \\ y_4\,z_3 \\ y_6\,z_1 \\ y_2\,z_5 \\ y_1\,z_6 \\ y_3\,z_4 \end{pmatrix} . \tag{27}$$

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